A Tour of JMP Reliability Platforms and Bayesian Methods for Reliability Data

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Abstract

JMP is a comprehensive, visual, and interactive statistical discovery software with a carefully curated graphical user interface designed for statistical discovery. The software is packed with traditional and modern statistical analysis capabilities and many unique innovative features. The software hosts several suites of tools that are especially valuable to the DATAWorks's audience. The software includes suites for Design of Experiments, Quality Control, Process Analysis, and Reliability Analysis. JMP has been building its reliability suite for the past fifteen years. The reliability suite in JMP is a comprehensive and mature collection of JMP platforms. The suite empowers reliability engineers with tools for analyzing time-to-event data, accelerated life test data, observational reliability data, competing cause data, warranty data, cumulative damage data, repeated measures degradation data, destructive degradation data, and recurrence data. For each type of data, there are numerous models and one or more methodologies that are applicable based on the nature of data. In addition to reliability data analysis platforms, the suite also provides capabilities of reliability engineering for system reliability from two distinct perspectives, one for non-repairable systems and the other for repairable systems. The capability of JMP reliability suite is also at the frontier of advanced research on reliability data analysis. Inspired by the research by Prof. William Meeker at Iowa State University, we have implemented Bayesian inference methodologies for analyzing three most important types of reliability data. The tutorial will start with an overall introduction to JMP's reliability platforms. Then the tutorial will focus on analyzing time-to-event data, accelerated life test data, and repeated measures degradation data. The tutorial will present analysis of these types of reliability data using traditional methods, and highlight when, why, and how to analyze them in JMP using Bayesian methods.

Biographies

Peng Liu is a Principal Research Statistician Developer at JMP Statistical Discovery LLC. He holds a Ph.D. in statistics from NCSU. He has been working at JMP since 2007. He specializes in computational statistics, software engineering, reliability data analysis, reliability engineering, time series analysis, and time series forecasting. He is responsible for developing and maintaining all JMP platforms in the above areas. He has a broad interest in statistical analysis research and software product development.

Introduction

JMP's modern reliability data analysis capabilities was initiated by the pioneering work of the company founder John Sall as early as in JMP 3. By the time the author took on the task of expanding the suite of products in 2007, JMP had four platforms that include Survival, Parametric Survival, Recurrence Analysis, and Proportional Hazards. In the last fifteen years, JMP added over a dozen new platforms, to fulfill the need of reliability engineers, and meanwhile stay on the frontier of reliability data analysis method's

research. Our achievements were propelled by our collaboration with Prof. William Meeker at Iowa State University. During our development, we constantly consulted Prof. Meeker on the directions of future development, and the rigorousness of methodologies' implementation. The books by him and his co-authors are the most important references of our development work.

JMP's reliability platforms now cover three areas: reliability data analysis, design of experiment for reliability, reliability engineering. The author is responsible for the development in data analysis and engineering areas. The suite of reliability platforms has made JMP a leading software vendor in the software market for reliability analysis and engineering.

This tutorial is developed to provide a brief overview about the JMP reliability platforms, with a keen emphasis on the Bayesian method applications among them. The tutorial will start with a method of classification of JMP reliability platforms. After the temptation of leaving the audience an impression about the vast available capabilities of JMP reliability platforms, the tutorial will turn to Bayesian method applications among them. It will include the following parts. First, the tutorial will establish a consensus on the data format for censored data in JMP software. Then, the tutorial will provide background information on Bayesian method. After that, the tutorial will go over four examples, which span three areas of applications.

Classifications of JMP Reliability Platforms

Reliability products in JMP can be categorized in different ways. One is by a characteristic of physical products that individual JMP reliability platforms are designed to analyze. This characteristic is whether a physical product is repairable or not. By such, these are the two lists.

Platforms for Non-repairable Systems	Platforms for Repairable Systems
Life Distribution	Recurrence Data Analysis
Survival	Reliability Growth
Reliability Forecast	Repairable Systems Simulation
Fit Life by X	
Parametric Survival	
Cumulative Damage	
Degradation	
Destructive Degradation	
Repeated Measures Degradation	
Reliability Life Plan	
Reliability Demonstration Plan	
ALT Design	
Reliability Block Diagram	

Table 1 JMP Reliability Platforms

Although there are many platforms, seemingly for different purposes, many of them lead to answers to a common question, which is the probability that a system, vehicle, machine, device, and so on, will perform its intended function under encountered operating conditions, for a specific period of time (Meeker and Escobar 1998, 2022). With an answer to that question, subsequent tasks can be carried out, such as warranty forecasting, system reliability design, etc.

The necessity of having variety of platforms is due to the number of different circumstances that different types of data are available, for which different statistical methodologies are required. From this perspective, some of JMP reliability products can be categorized in the following way.

Objective	Platform Name(s)	Applicable Circumstance
Obtain Failure Probability of a System	Life Distribution	Time-to-event data
	Fit Life by X	Time-to-event data with one time-invariant factor
	Cumulative Damage	Time-to-event data with one time-varying factor
	Parametric Survival	Time-to-event with multiple time-invariant Factors
	Repeated Measures Degradation	Degradation data from non-destructive experiments or observation
	Destructive Degradation	Degradation data from destructive experiments or observation
Track Reliability of a Process	Recurrence Data Analysis	Counting process data with multiple covariates
	Reliability Growth	Counting process data for project development process.

Table 2 Applicable Circumstances of JMP Reliability Platforms

Within individual JMP reliability products, there might be several different models, and the following is a list which describes what are different models within a reliability platform for some platforms.

Platform	Models
Life Distribution	Numerous distribution types; Competing Cause; Mixture
Fit Life by X	Numerous distribution types and different acceleration mechanisms
Cumulative Damage	
Repeated Measures Degradation	Different degradation mechanisms
Destructive Degradation	Different degradation mechanisms
Recurrence Data Analysis	Different point process types
Reliability Growth	Different application-oriented point process types

Table 3 Modeling Varieties within Platforms

To provide a taste of the variety of reliability products in JMP, Figure 1 is a collection of some screenshots from individual platforms.

In the end, there are different statistical methodologies in several platforms for the same type of data, same type of models. Maximum likelihood method is the dominant approach to address almost all the need for analyzing reliability data. Bayesian methods have been implemented in three platforms for all or part of the supported models. The three platforms are: Life Distribution, Fit Life by X, and Repeated Measures Degradation. The next section starts our emphasis on Bayesian method applications in JMP for reliability data analysis.

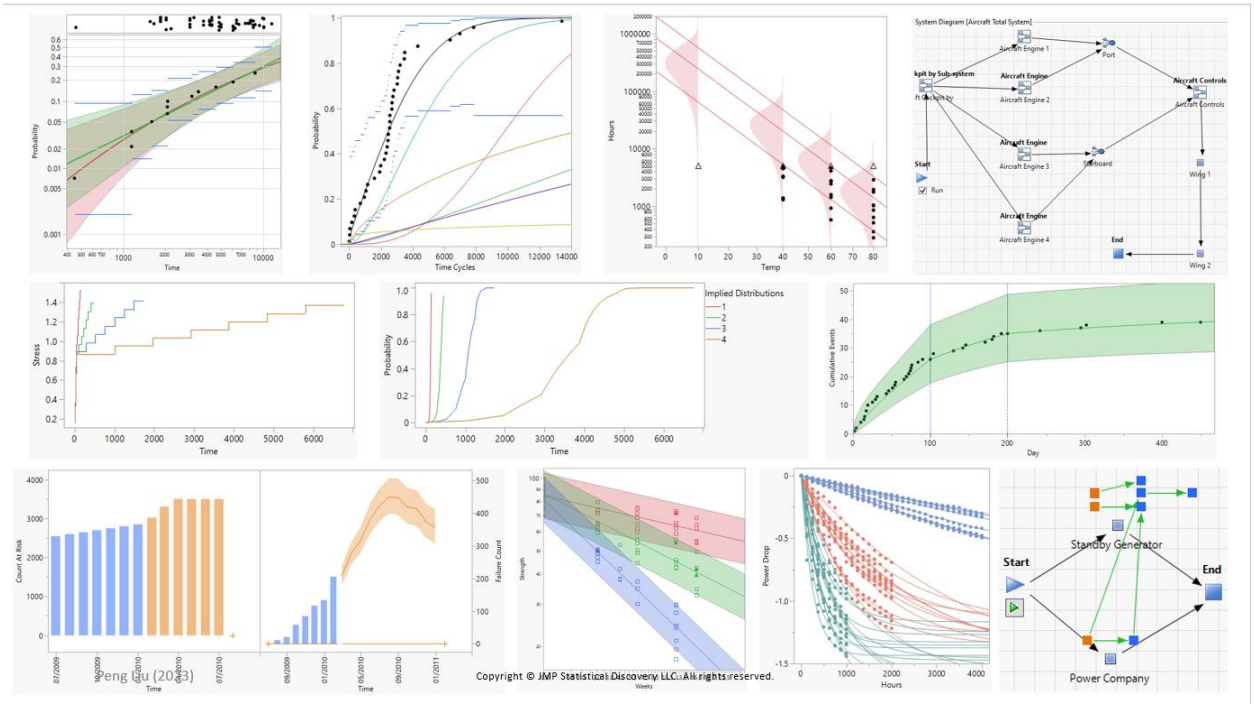


Figure 1 Screenshots from JMP Reliability Platforms

Bayesian Methods for Reliability Data Analysis

Traditionally, Bayesian methods were thought of as alternatives to frequentist's approaches, mainly the maximum likelihood approach. Experts in Bayesian methods were usually specially trained separately from frequentist statisticians. The mathematical requirements for obtaining such skills were high, due to the necessity of derivations of highly sophisticated models. The practicality of applying Bayesian methods was low because many solutions lack closed form formulas and require computational tools that were not available. Even though some computational tools became available, the computers were not powerful enough for non-trivial problems.

In the context of reliability data analysis, Bayesian methods are needed naturally in circumstances when failures are scarce. There is a well-known method called Weibayes (Nelson 1985), which is arguably non-Bayesian at all, but reminds us of the need of Bayesian approaches when there is little information in data. Under the Bayesian framework, additional information can be brought into modeling besides what is already in the data, by such one can draw more useful conclusions.

Besides the circumstances when data is scarce, Bayesian methods for reliability data analysis are more attractive than frequentist's approaches when the models are more complex, non-linear, or both. In such situations, frequentist's approaches face increasing challenges in inference which require expensive cost in method development and computational requirements. Meanwhile, due to lack of uniqueness of frequentist's approaches given a specific model, the interpretation of results becomes quickly confusing. On the other hand, Bayesian approaches address all the issues with ease.

What remains challenging for Bayesian approaches is the computational cost in terms of time. Because it has become a common practice to use so called Markov chain Monte Carlo (MCMC) to conduct Bayesian inference, and MCMC methods require lengthy simulations. However, with the increasing power of

computers and accessibility to modern software, such a challenge is no longer an absolute barrier for conducting Bayesian analysis.

Consensus on Censored Data Format in JMP

Censored data is probably the first factor that separates reliability data analysis from other types of data analysis. Many reliability data records time-to-event observations. Some of such observations even have their special names in different contexts. For example, here are some more common words that describe censored observations: survived, suspended, runout, which may sound more familiar to some people.

The three words all mean right-censoring type, which indicates that failure does not occur at the observation time. Other than that, there are left-censoring and interval-censoring types, which indicate failure has occurred, but the exact time is unknown. For left-censoring, it is a type which indicates the failure occurs before the observation time. And for interval-censoring, it is a type which indicates the failure occurs within a time interval. In addition to three types of censoring, exact failure time observations observe failure time precisely. For any given censored data set, there might be one or more types of observations.

JMP reliability platforms expect one of the two types of data formats. The first is known as the right-censored data format. And the other is known as the two-column censored data format. For the right-censored data format, the time-to-event data is represented by one or two columns. If it is represented by one data column, the column records exact failure times. If it is represented by two data columns, the first column records time-to-event, and the second column records whether corresponding events are failures or right-censoring observations. For example, the partial screenshot on the left in Figure 2 is for Appliance sample data in JMP, and the column "Time Cycles" records exact failure times. The partial screenshot on the right in Figure 2 is for Fan sample data in JMP, and the column "Time" records time-to-event, and the column "Censor" indicates whether the event is a failure or a right censoring observation.

	Time Cycles	Cause Code
1	11	1
2	2223	9
3	4329	9
4	3112	9
5	13403	0
6	6367	0
7	2451	5
8	381	6
9	1062	5
10	1594	2

	Time	Censor
1	450	Failed
2	460	Censored
3	1150	Failed
4	1150	Failed
5	1560	Censored
6	1600	Failed
7	1660	Censored
8	1850	Censored
9	1850	Censored
10	1850	Censored

Figure 2 Appliance Sample Data and Fan Sample Data

Therefore, using the right-censored data format, one can represent two types of censoring schemes. One is exact failure data. And the other is a mix of failures and right censored observations. Is it possible that all observations are right-censored? Yes, it is possible, and in that case, maximum likelihood estimate does not exist. And that is one of several scenarios that have been discussed in Nelson 1985, in which the Weibayes method was proposed.

The second censored data format that JMP recognizes is the two-column format, in which case both columns record time values. And different arrangements of the two time values in a row indicate individual censoring schemes. There are four valid arrangements. The first arrangement is that two time values are valid and equal. That indicates an exact failure observation. The second arrangement is that the value on the left is non-missing, and the value on the right is missing. That indicates a right censored observation. The third arrangement is that the value on the left is missing, and the one on the right is non-missing. That indicates a left-censored observation. The last arrangement is that the two time values are valid (non-missing) and unequal, with the left value is less than the value on the right. That indicates an interval censored observation. Figure 3 is a partial screenshot of the Microprocessor Data in JMP.

	start time	end time	count
1	•	6	6
2	6	12	2
3	24	48	2
4	24	•	1
5	48	168	1
6	48	•	839
7	168	500	1
8	168	•	150
9	500	1000	2
10	500	•	149

Figure 3 Microprocess Data

To configure JMP launch dialog, use one of the following three types to specify for corresponding censoring schemes and data formats.

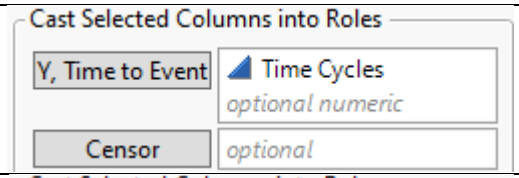
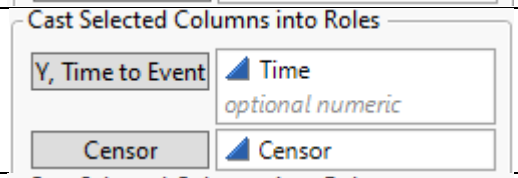
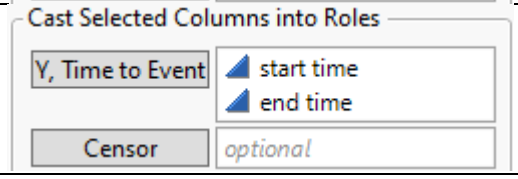
Specify launch dialog for exact failure data.	
Specify launch dialog for right-censored data with a Censor indicator column.	
Specify launch dialog for arbitrary censoring schemes with two-column format data.	

Figure 4 Launch Dialog Configurations for Different Data Format

Bayesian Methods

This section attemps to lay a fundation for the remaining discussion. It does not attempt to be even an introduction to the Bayesian methods. The information here serves as the necessity to operate the Bayesian model fitting for reliability data analysis in JMP. And it also serves as a succint guide for directions if one is interested in learning more.

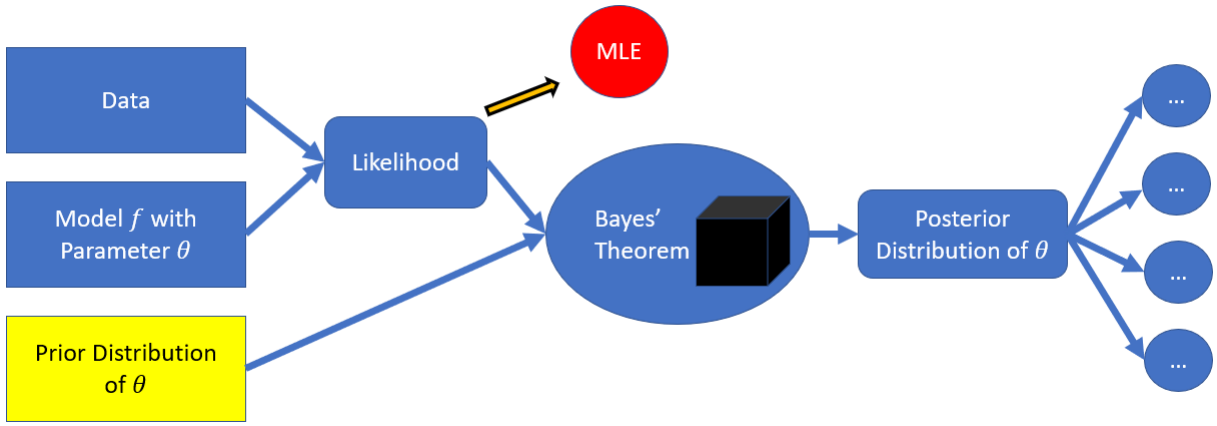


Figure 5 Bayesian Methods

Figure 5 depicts the critical elements in the Bayesian method framework. As you may see, the approach is high related to frequentist's maximum likelihood method. For maximum likelihood method, one needs data and a model with parameter which will form a likelihood function which is a function of one or more parameters. Then one would maximize the likelihood function over the parameters. And the parameter values that maximizes the function is known as the maximum likelihood estimate (MLE.) For Bayesian method, one needs to supply prior distribution for parameters. Prior distribution is a function of the parameters as well. Combining likelihood and prior distribution, one then apply the Bayes' theorem, and obtain the corresponding posterior distribition of the parameters. One can then derive subsequent quantities from the posterior distribution, which include poterior median, posterior mean, credible intervals, etc.

We will assume that the choice of model is clear. The first important element for conducting a Bayesian modeling is to specify prior distribution for parameters. There are at least several technical terms that one may need to know, which includes non-informative prior, informative prior, and weakly informative prior. There are other important technical terms for types of prior, but they are irrelevant at the moment for using the capabilities that are available in JMP reliability platforms. The next several paragraphs describe the three types of prior.

The non-informative prior is a very common term. But it seems to be easily misunderstood, and its sophistication is often overlooked, by many. In another word, whether a prior is absolutely non-information requires careful examination. The remaining discussion will avoid examine whether a prior is non-informative.

The remaining discussion will only refer to weakly informative and informative priors. To distinguish them, the author would like to point out that the difference between the two is the amount of information contained in the prior, which will influence the posterior distribution differently. By using a weakly

informative prior, the prior should have little influence on the posterior. Or in another work, if one changes the weakly informative prior substantially, but still maintain the weakly informative characteristic of the prior, the posterior distribution should not change substantially. By using an informative prior, however, one would expect the posterior to be sensitive to the choice of the prior. Or in another workd, if one change the informative prior, it is likely to lead to noticeable changes in posterior.

Bayesian Computation

The black box in Figure 5 represents the important part in Bayesian methods, which is the computational part. In practice, posterior distributions rarely have closed form, and require simulation. In the author's practice, there are mainly four schools of general purposed simulation-based approaches. They are:

1. Rejection sampling algorithm,
2. Gibbs sampling algorithm,
3. Metropolis-Hastings algorithm,
4. Hamiltonian Monte Carlo algorithm.

The first three are implemented and applied within JMP reliability products at various places. The last method requires substantially more effort than others, such that the author has not attempted to implement yet.

Except for the first method, the remaining three methods require non-trivial software development skills. And general purposed implementations are rather sophisticated. Noticeable software includes WinBUGS and its variations for Gibbs sampling algorithm, SAS Proc MCMC for the Metropolis-Hastings algorithm, and STAN for Hamiltonian Monte Carlo algorithm.

Gibbs sampling algorithm and Metropolis-Hastings algorithm are mainstream approaches for a very long time. Within the last ten years or so, Hamiltonian Monte Carlo algorithm appears to be the most popular approach, due to the development of STAN (https://mc-stan.org/), and its access through well-known open-source software R (https://www.r-project.org/).

Regardless of which software to choose, there are some common considerations. They are:

1. Parameterization of the model,
2. Configuration of prior distributions,
3. Efficiency and effectiveness of the simulation,
4. Diagnostics of the posterior samples.

Parameterization is usually not a concern in general in Bayesian method literature. For the applicability of Bayesian method, however, parameterization is important. It is desirable to have a parameterization, such that it is easier to solicit prior from engineers. Such parameterization usually contains parameters that have practical meanings to engineers, while the information on individual parameters is independent from one another. With such parameterization, configuration of prior distributions is relatively easy for any given software.

All simulation-based approaches mentioned previously have efficiency concerns. A rejection sampling algorithm is not efficient if the rejection rate is too high. A Gibbs sampling algorithm is not efficient if some parameters are highly correlated. A Metropolis-Hastings algorithm and a Hamiltonian Monte Carlo

algorithm may not be efficient and effective if the posterior samples are highly autocorrelated. By knowing these concerns, one may prefer one method to another.

In the end, like many other statistical inference methods, it requires one to be familiar with diagnostics and be aware of possible issues before making conclusions and conducting further analysis using the posterior samples.

The next four sections go through four examples to illustrate how to use Bayesian methods to analyze three types of reliability data in JMP.

Rocket Motor

The rocket-motor (Meeker, et.al. 2022 Chapter 10, Olwell and Sorell 2001) records the status of 20000 missiles in US Navy's inventory. Each missile has a rocket motor. Over time, 1940 missiles had been used. And among them, three had catastrophic launch failures. Missiles in the inventory have age from one year up to about 18 years.

Engineers and scientists inspected the failures and identified an unexpected failure mode. They are concerned that the failure mode may induce unexpected failure rate increase during wear out period. There can be a fix by replacing the aging motors, but highly expensive, to be applied to large scale of the stockpile. They would like to know the number of missiles that require such fix, based upon the estimated failure probability, given age.

	Years Lower	Years Upper	Count	Censoring Indicator
1	1	•	105	Right
2	2	•	164	Right
3	3	•	153	Right
4	4	•	236	Right
5	5	•	250	Right
6	6	•	197	Right
7	7	•	230	Right
8	8	•	211	Right
9	9	•	124	Right
10	10	•	90	Right
11	11	•	72	Right
12	12	•	53	Right
13	13	•	30	Right
14	14	•	14	Right
15	15	•	5	Right
16	16	•	3	Right
17	0	16.5	1	Left
18	0	8.5	1	Left
19	0	14.2	1	Left

Figure 6 Rocket Motor Data

Figure 6 is a screenshot of the data. The first 16 rows record uses that were normal. And it uses two-column format to indicate right-censoring situation for these missiles. For example, 106 missiles were used one year after manufacturing. And the two-column format indicates they are right-censored, which means they did not failure at one year. The last three rows record three failures. And the two-column format indicates they are left-censored, which means they failed sometime before use at the corresponding recorded time. The Censor Indicator column comes with the original data, which indicates left- and right-censoring. Such format is not supported in JMP, and the column will not be used in subsequent analysis.

To start analyzing this data in JMP, first open the data table, then from JMP menu, select Analyze > Reliability and Survival > Life Distribution, to bring up the launch dialog. Then configure the dialog as follows.

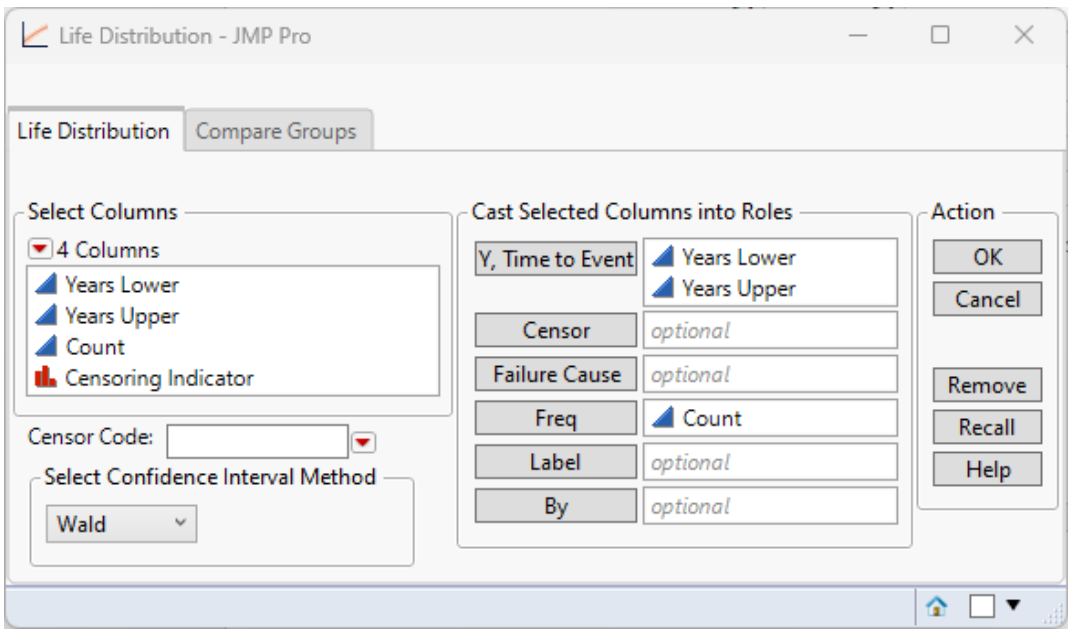


Figure 7 Life Distribution Launch Dialog

The default report shows the nonparametric estimate of the failure probability function. And a Weibull distribution is desired to fit to the data. To fit a Weibull, use the check box or menu item to activate the fit, then change the axis scales to reflect a Weibull probability paper.

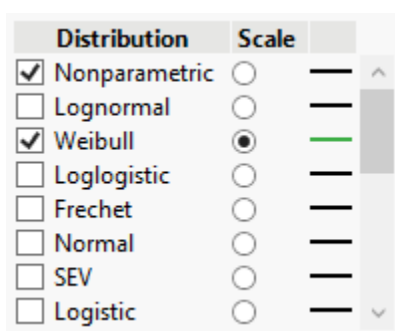


Figure 8 Life Distribution Choices

The next step is to look at the estimated failure probability at 20 years. The target is to have no more than one percent of failure probability at 20 years. Here, a quick look at the distribution profiler will raise the concern about the severity of the situation.

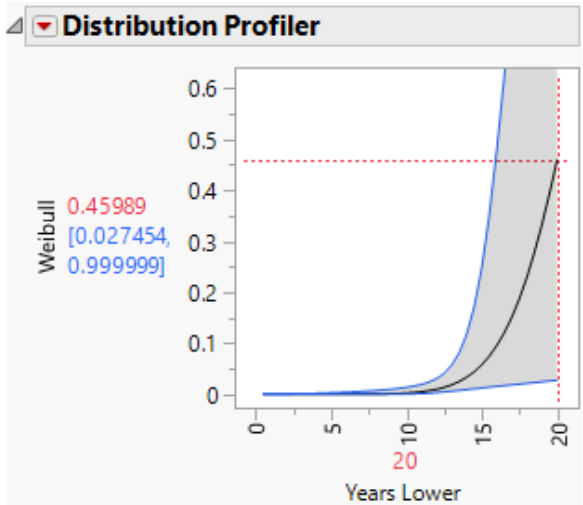


Figure 9 Distribution Profiler

There are at least two concerns. First, the failure probability at 20 years is alarmingly high. Second, the confidence interval of the estimate is extremely wide. The confidence interval in the distribution profiler is the Wald type interval, which is less trustworthy for this data with few failures. But using a Likelihood confidence interval won't bring better news.

Estimate Probability

Years Lower	Failure Probability	Failure Probability 95% Confidence Interval (Wald)		Failure Probability 95% Confidence Interval (Likelihood)	
		Lower	Upper	Lower	Upper
20	0.45989045	0.02745352	0.9999988	0.02345631	0.9999957

Figure 10 Likelihood Confidence Interval

Fortunately, engineers and scientists have some prior knowledge about the plausible range of the Weibull shape parameter. MLE is about 8, and that is unrealistically large according to engineers and scientists.

Parametric Estimate - Weibull

Parameter	Estimate	Std Error	95% Confidence Interval (Wald)	
			Lower	Upper
location	3.055358	0.2162706	2.631476	3.479241
scale	0.123058	0.0480368	0.028908	0.217209
Weibull α	21.228791	4.5911625	13.894260	32.435089
Weibull β	8.126236	3.1721454	4.603869	34.592771
Mean	20.007199	3.9737989	13.555700	29.529128

Figure 11 Maximum Likelihood Estimate

And the range (1, 5) is much more plausible. The next shows how we incorporate that information into the model.

Bayesian Estimation - Weibull

Specify range for quantile and scale.

Name	Prior Distribution		Value		Value		Value
Quantile	Normal	Probability	0.1000	Lower 99% Limit	5.0000	Upper 99% Limit	400.0000
Weibull β	Normal			Lower 99% Limit	1.0000	Upper 99% Limit	5.0000

Number of Monte Carlo Iterations 5000

Random Seed 1682455648

Show Prior Scatter Plot ☐

Overlay Likelihood Contour ☐

Fit Model

Figure 12 Prior Distribution of Weibull Model for Rocket Motors

The configuration in Figure 12 assigns independent prior distributions to the Weibull shape parameter, and 10% quantile. The priors are both Normal distributions. But they are specified in a way, such that it is easier to solicit information. The well-known parameters of a Normal distribution are location parameter μ and scale parameter σ. It is, however, not intuitive to transfer knowledge directly to these two parameters. It is the range of plausible values that is more common to know. Therefore, the software provides an interface which solicits the lower and upper end points of the 99% interval of the distribution.

In Figure 12, the 99% interval of the Normal prior for Weibull shape parameter is 1 and 5. The range (1, 5) is rather informative. The software will translate the information to get values for μ and σ. During sampling, the effective prior is a truncated Normal, because the quantity needs to be positive. Similarly, the 99% interval for the Normal prior for the 10% quantile is (5, 400), or in another word, 10% of units are very likely to fail between 5 years and 400 years. That is a rather wide span. In such case, we are assigning a so-called weakly informative prior to the quantity.

You may notice that the software does not list the Weibull scale parameter for soliciting prior. This is related to the choice of parameterization that was mentioned previously. There are two reasons that we prefer this parameterization, which includes a quantile and Weibull shape parameter. The first reason is that the quantile has more practical meaning than the scale parameter, such that engineers and scientists have better knowledge about the quantile. The second reason is that the information on the quantile and the shape parameter is more independent, such that providing separate prior distributions on quantile and shape parameter respectively is very plausible. Now we click “Fit Model” button and generate a report (Figure 13).

The top of the report indicates that the simulation method that draws samples from the posterior distribution is Simple Rejection. If the rejection rate becomes high, the software will automatically switch to the Metropolis-Hastings random walk algorithm. The Priors outline node section is a copy of the prior specification. The Posterior Estimates outline node section reports the summaries of relevant quantities. The distribution that we fit is Weibull. The posterior estimates report shows posterior summaries of Quantile and Weibull shape, the two quantities that we specify prior distributions. In addition, we provide

posterior summaries on the location parameter μ and scale parameter σ. Subsequent computation is usually easier by using the (μ, σ) parameterization.

Bayesian Estimates - Result 1

Method: Simple Rejection

Priors

Name	Prior Distribution		Value		Value		Value
Quantile	Normal	Probability	0.1000	Lower 99% Limit	5.0000	Upper 99% Limit	400.0000
Weibull β	Normal			Lower 99% Limit	1.0000	Upper 99% Limit	5.0000

Random Seed 1682455648

Posterior Estimates

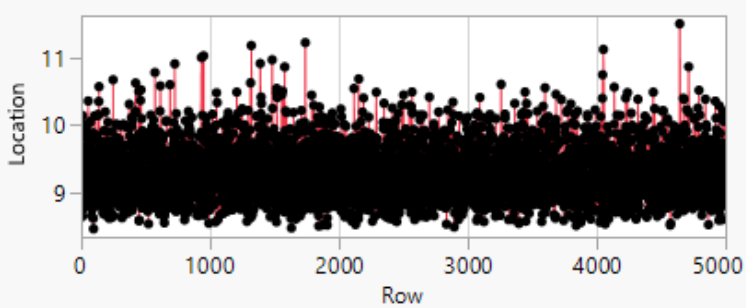
Export Monte Carlo Samples

Parameter	Median	95% Credible Interval Lower	95% Credible Interval Upper	Mean	StdDev
Location	4.899672	3.597227	7.36932	5.136913	1.110363
Scale	0.416499	0.231262	0.845214	0.455635	0.169745
Quantile (p=0.1)	52.28085	21.06564	263.5598	82.32904	68.95252
Weibull β	2.400813	1.182545	4.322674	2.497872	0.875958

Figure 13 Bayesian Estimation of Weibull for Rocket Motors

Notice there is an Export Monte Carlo Samples hyper link on the side, which is to remind user to inspect the posterior samples. Click the hyper link, one will get the posterior samples in a table, and one should at least look at the time series plot, or more commonly known as the trace plot in Bayesian literature. The task is relatively trivial, if one knows how to launch a platform, and knows where the Time Series menu item is under the Analyze menu. In particular, we want to inspect whether the posterior is highly autocorrelated. This will not be any issue for rejection sampler. So, we see the autocorrelation is zero.

Time Series Location



Mean	9.3026413
Std	0.3672732
N	5000
Zero Mean ADF	-2.015447
Single Mean ADF	-72.29706
Trend ADF	-72.29998

Time Series Basic Diagnostics

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.0221	2.4360	0.1186	1	-0.0221
2	0.0336	8.0820	0.0176*	2	0.0331
3	-0.0173	9.5746	0.0226*	3	-0.0158
4	-0.0024	9.6041	0.0477*	4	-0.0042

Figure 14 Time Series and Autocorrelation

After inspecting the posterior samples, we can conclude that there are no abnormal issues in the posterior, and we can proceed to make inference about failure probability at 20 years. Towards the end of the report for this model, there are two profilers. We will use the Distribution Profiler to get an estimate of the failure probability at 20 years. The estimate is just about one percent, and the credible interval ranges from 0.001752 to 0.082136.

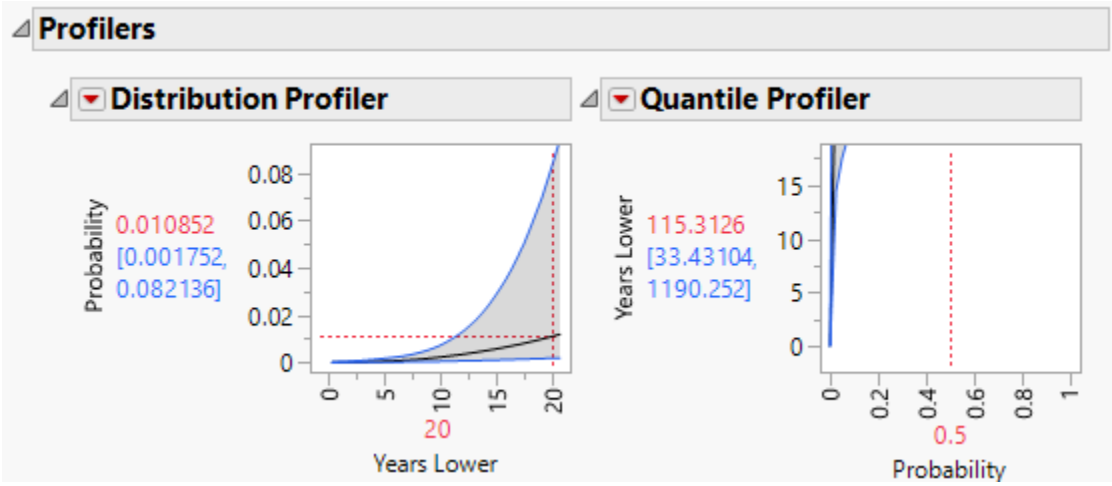


Figure 15 Profilers

The result indicates that the situation may not be as bad as what the maximum likelihood estimates imply, but it is also not as optimistic as the engineers and scientists wish the situation to be. Ideally, they wish the failure probability at 20 years is no more than one percent. But here the upper end of the credible interval exceeds that threshold.

IC Device

The IC Device data (Meeker, et. al. 2022. Chapter 19) is from a temperature accelerated life test on some integrated circuit board. Following is the screenshot of the data.

	HoursL	HoursU	Status	Count	DegreesC
1	1536	•	Right	50	150
2	1536	•	Right	50	175
3	96	•	Right	50	200
4	384	788	Interval	1	250
5	788	1536	Interval	3	250
6	1536	2304	Interval	5	250
7	2304	•	Right	41	250
8	192	384	Interval	4	300
9	384	788	Interval	27	300
10	788	1536	Interval	16	300
11	1536	•	Right	3	300

Figure 16 IC Device Data

The experiment was conducted under five different temperature levels. Failures only occurred at 250°C and 300°C, the two high temperature levels. Observations at other lower temperature levels are right censored. It is known that, in this case, acceleration follows the Arrhenius relationship. The interest is in the activation energy, and fraction of failures at 100,000 hours at 100°C, which is over 10 years. The system is highly reliable.

Furthermore, it was discovered that the failures at 300°C are due to a failure mode which won't happen under the normal use condition. Therefore, the observations at 300°C must be discarded to avoid concluding a wrong model, which would lead to wrong conclusions.

After discarding the bad observations using exclusion as follows, we launch Fit Life by X platform to analyze the data.

	HoursL	HoursU	Status	Count	DegreesC
1	1536	•	Right	50	150
2	1536	•	Right	50	175
3	96	•	Right	50	200
4	384	788	Interval	1	250
5	788	1536	Interval	3	250
6	1536	2304	Interval	5	250
7	2304	•	Right	41	250
8	192	384	Interval	4	300
9	384	788	Interval	27	300
10	788	1536	Interval	16	300
11	1536	•	Right	3	300

Figure 17 IC Device Data with 300°C Group Excluded

To launch Fit Life by X platform, select Analyze > Reliability and Survival > Fit Life by X. Then configure the dialog as follows:

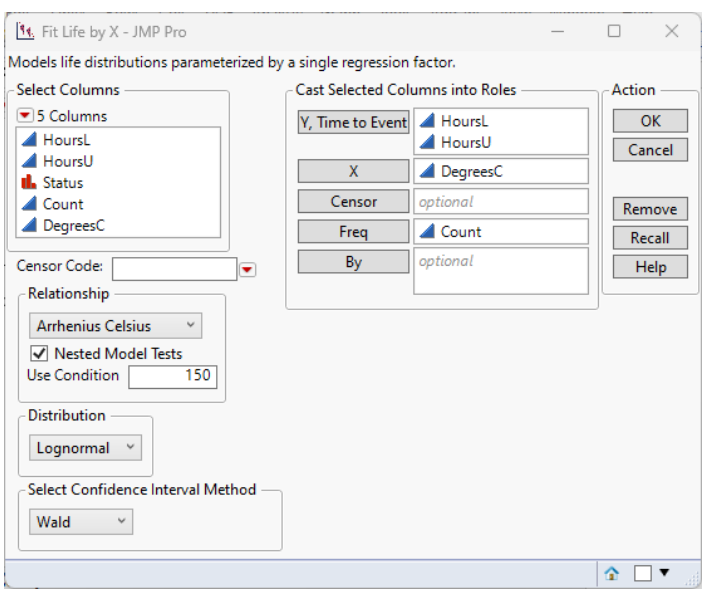


Figure 18 Fit Life by X Launch Dialog

Click OK, a report is generated, which fits an Arrhenius-Lognormal model. The report includes numerous information, which provides different perspectives on the same model. The scatterplot below overlays data (markers), fitted Arrhenius relationship (three straight lines), and estimated densities at multiple temperature levels. The slope of the straight lines is the activation energy of interest.

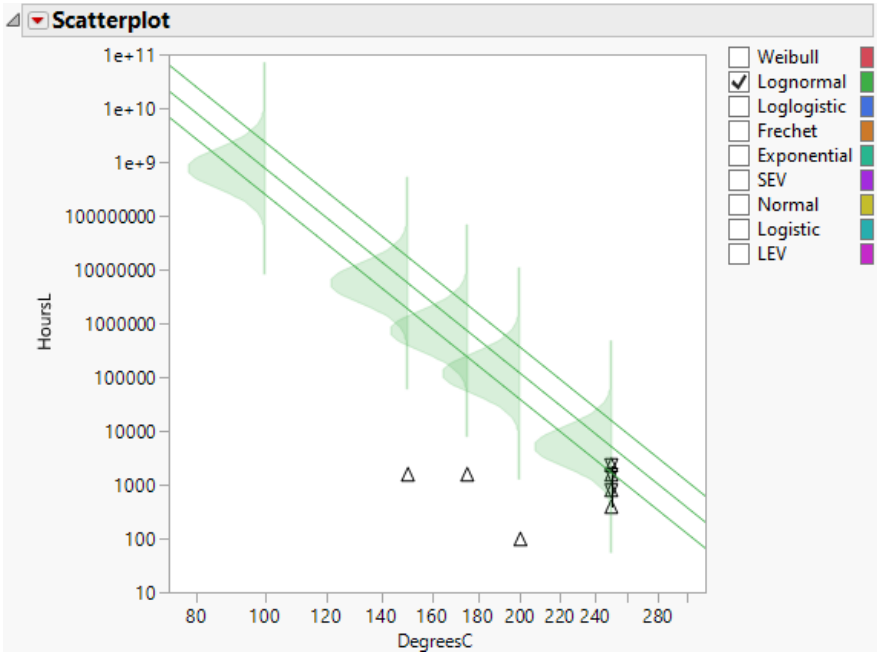


Figure 19 Scatterplot in Fit Life by X

The scatterplot indicates that the fractional failure at 100,000 hours at 100°C is negligible. There are, however, issues that indicate that the result is not trustworthy.

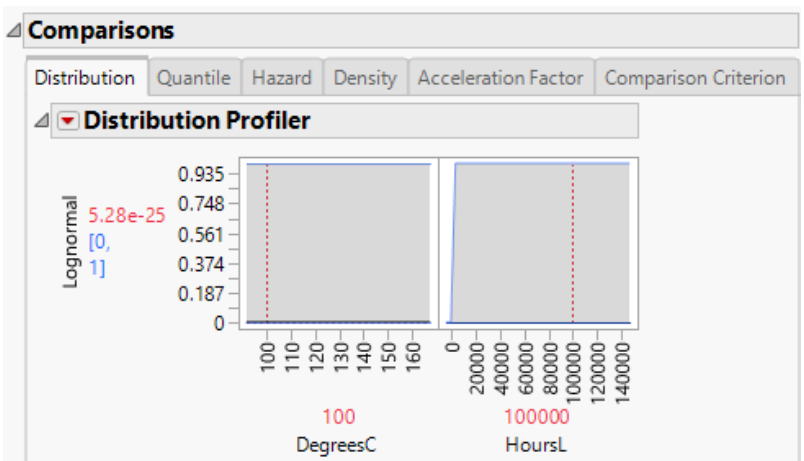


Figure 20 Distribution Profiler in Fit Life by X (IC Device MLE)

First is that the confidence interval is extremely wide for the estimated fractional failure at 100,000 hours at 100°C. Second is that the confidence intervals are extremely wide for the parameter estimates, which is the fundamental reason why the confidence interval is wide for the fractional failure. All this is because the data contains little information about the activation energy.

With the data alone, the result is not very useful. In practice, if engineers know the failure mode, they may have knowledge about the activation energy. In this example, we apply the following settings to configure our Bayesian model.

Lognormal Results

Statistics | Distribution | Quantile | Hazard | Density | Acceleration Factor | Custom Estimation | Bayesian Estimates

Specify range for quantile, slope, and scale.

Name	Prior Distribution	DegreesC	Probability		Value		Value
Quantile	Lognormal	250.0000	0.1000	Lower 99% Limit	100.0000	Upper 99% Limit	32000.00
β1	Lognormal			Lower 99% Limit	0.8000	Upper 99% Limit	0.9500
σ	Lognormal			Lower 99% Limit	0.0625	Upper 99% Limit	16.0000

Number of Monte Carlo Iterations 5000

Random Seed 1

Fit Model

Figure 21 Prior Distribution of Bayesian Model for IC Device

The software asks for information about three quantities that engineers may have prior information. And they are very likely to be independent from one another. As we have noted, the prior distributions are specified by using the ends of 99% interval. For this example, we use Lognormal for all three quantities. The prior on β_1, the activation energy, is rather informative. And the priors on the other two quantities are weakly informative. To determine whether a prior distribution is informative or weakly informative, one can make judgement based on the range of the interval. That might be another advantage of using intervals to specify prior distributions. One may also want to conduct sensitivity studies to see how posterior distributions are influenced by prior distributions. If prior distributions have little influence on the posterior distributions, then it should be safe to assume the prior distributions are weakly informative.

Now click "Fit Model" button to continue the analysis. Like what is shown in the Rocket Motors example, a Bayesian model result (Figure 22) in JMP Fit Life by X platform indicates what sampling method was used to produce the result. In this example, the method is the rejection-based sampling method. As the rejection rate increases, the software will automatically switch to Metropolis-Hastings random walk MCMC algorithm and produce subsequent results.

The next section in the report is a copy of prior distribution settings. And the Posterior Estimates outline node summarizes the posterior samples. And there is also a hyper link for one to inspect and export posterior samples.

Bayesian Estimates - Result 1

Method: Simple Rejection

Priors

Name	Prior Distribution	DegreesC	Probability		Value		Value
Quantile	Lognormal	250.0000	0.1000	Lower 99% Limit	100.0000	Upper 99% Limit	32000.00
β1	Lognormal			Lower 99% Limit	0.8000	Upper 99% Limit	0.9500
σ	Lognormal			Lower 99% Limit	0.0625	Upper 99% Limit	16.0000

Random Seed 1

Posterior Estimates

Export Monte Carlo Samples

Parameter	Median	95% Credible Interval Lower	95% Credible Interval Upper	Mean	StdDev
β0	-10.605	-12.0491	-8.96666	-10.5945	0.783242
β1	0.871528	0.817547	0.931408	0.872172	0.029077
σ	0.98268	0.572253	1.880291	1.048132	0.338548
Quantile (p=0.1)	1662.555	1018.841	2504.074	1691.379	379.4708

Figure 22 Bayesian Model Result in Fit Life by X

After seeing posterior samples, we can conclude that there are no abnormalities. And we can proceed to draw conclusions about the real interest, which is failure probability at 100,000 hours at 100°C. A look at the distribution profiler at the end of the report, with a couple of adjustments to the settings of the two axes, we get the following result.

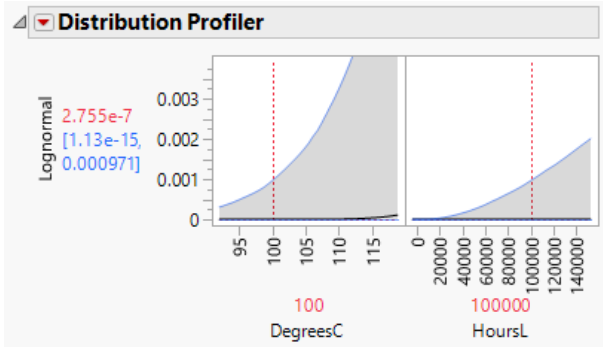


Figure 23 Distribution Profiler in Fit Life by X (IC Device Bayesian Estimation)

Compared to the extremely wide confidence intervals from maximum likelihood method, here the credible interval is much more reasonable given the prior information from engineers.

This section and the previous section have demonstrated how Bayesian methods are extremely useful when the information in data is limited. In such cases, an informative prior on some quantities of the model can lead to more useful results that are both trustworthy and useful so that engineers can act upon. The next couple sections will demonstrate a different scenario, in which case there is plenty of information in the data, but the model is also complex.

Introduction to Repeated Measures Degradation Model

The nature of data for this type of analysis consists of multiple units, systems, or subjects. Multiple measurements are taken from the individual units over a period. Meanwhile, there may or may not be extra factors accompanied. Examples of factors include temperature, humidity, voltage, etc.

The type of models that are relevant consists of two types of parameters. The first type of parameters can be named as population parameters. And the second type of parameters can be named as unit parameters. Population parameters are applicable to all units, while unit parameters are only applicable to corresponding units. And unit parameters are governed by a portion of population parameters.

Such data and models are surprisingly common in a diverse range of fields, including agriculture, economics, medical, clinical, and reliability, etc. The models can be classified in multiple ways from different perspectives. One way of classification is based on whether a model is linear in parameter or not. There is a large body of research which focuses on linear models, from frequentist's perspective. And the methodologies are well established and widely accepted among engineers and scientists. There, however, are challenges when a model is not linear in parameters. Without going further into the technical details, we would like to point out fitting such a model, regardless of linear or non-linear, requires integrating out the unit parameters. And integration is not a straightforward task in many situations. In many situations, approximation is applied, such that subsequent analyses would face a challenge for interpretation. To emphasize, frequentist's approach heavily relies on the large sample theory. And in various situations and steps, such dependence may draw criticism, about accuracy and uniqueness.

Meanwhile, Bayesian methods handle the integration with ease. And there is only one way to conduct the inference, which leads to one result, without ambiguity. The challenging part of the Bayesian approach is also computational, but as the computers become more powerful, and available software is accessible, the computational challenge is arguably less of an issue. We will illustrate a couple of examples of conduction Bayesian estimation in Repeated Measures Degradation platform in JMP.

Alloy A

The data records crack propagation over time on 21 specimens of a type of metal alloy. On each specimen, there are about a dozen crack length measurements measured at interval of every 10,000 cycles. And the measurements stop at around 120,000 cycles.

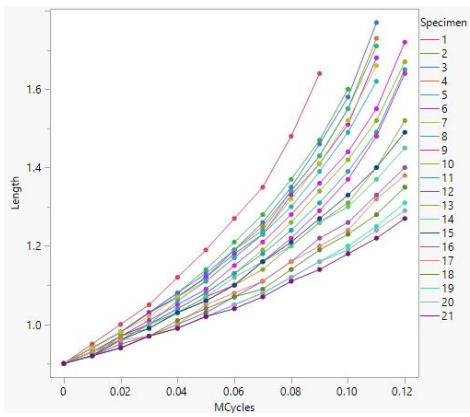


Figure 24 Alloy A Data

Figure 11 is a plot of the data. In the plot, individual measurements are drawn as markers. Measurements from the same unit are connected by lines. The unit of x-axis is million cycles, and the unit of y-axis is inch.

The propagation of the crack growth is governed by a theory known as the Paris' Law. The general form of the model is expressed as an ordinary differential equation, as follows:

$$\frac{d\mu(t)}{dt} = b_1 \cdot \left(\mu(t)\right)^{\frac{b_2}{2}}.$$

The differential equation has two solutions, depending on the value of parameter b_2. Here are the two solutions:

$$\begin{aligned} &\left(\mu_0^{\left(1-\frac{b_2}{2}\right)} + \left(1-\frac{b_2}{2}\right)\cdot b_1 \cdot t\right)^{\frac{2}{2-b_2}} &&, \quad if\ b_2 \neq 2; \\ &\mu_0 \cdot \exp(b_1 \cdot t) &&, \quad if\ b_2 = 2. \end{aligned}$$

There are two unit-parameters b_1 and b_2. And they are different from unit to unit. In another word, for every 21 units in the data, every unit's crack growth path is governed by its own pair of b_1 and b_2 values. Therefore, there are 21 b_1 and 21 b_2 parameters. Meanwhile, the randomness of b_1 and b_2 is governed by a distribution. In our example, the distribution is a bivariate normal distribution. A bivariate normal distribution has five parameters, which include two location parameters (μ_1, μ_2), two scale parameters (σ_1, σ_2), and one correlation coefficient parameter (ρ). Governed by a pair of b_1 and b_2 values, the crack growth path is not deterministic. In this example, we assume that measurement deviates randomly from

the path, and the random deviations are i.i.d. normal with zero mean and standard deviation σ_ϵ. The six parameters μ_1, μ_2, σ_1, σ_2, ρ, and σ_ϵ are population parameters.

The ultimate interest is in those six parameters, not in the unit parameters. The reason is that one needs those six parameters to describe the randomness of the crack growth path. Then a failure distribution for metal fatigue can be derived. To get estimates of those six parameters, a frequentist's approach needs to write down the likelihood, which is a function of all unit parameters and population parameters. Then one needs to maximize the likelihood with unit parameters integrated out. That is a challenging task to calculate the integral while trying to maximize it.

It is, however, relatively easy for a Bayesian approach. Arguably speaking, to engineers, they only need to specify prior distributions to the six parameters. And ideally, a posterior sampling algorithm will take care of the rest. That is an important use case of Bayesian inference, which is to explore a complex likelihood function. That is what the JMP Repeated Measures Degradation is designed to do by default. By default, the software provides a so-called weakly informative prior based on a pilot study about the data. By such, the posterior distribution is very likely proportional to the likelihood. Such that, we solve a complex problem with ease, with a solution which is close to what a frequentist method would produce, and such a frequentist method may not be available.

In the remaining section, we demonstrate how to estimate the parameters in this Paris model using the Bayesian method in JMP Repeated Measures Degradation platform. And, how to use the estimates to make further inference that leads to the failure distribution of metal fatigue.

The Alloy-A data has three columns. Length records the crack length in inches. Speciman indicates individual units. And MCycles is the number of cycles in million of cycles.

	Length	Specimen	MCycles
4	1.05	1	0.03
5	1.12	1	0.04
6	1.19	1	0.05
7	1.27	1	0.06
8	1.35	1	0.07
9	1.48	1	0.08
10	1.64	1	0.09
11	0.9	2	0
12	0.94	2	0.01
13	0.98	2	0.02

Figure 25 Alloy A Data

To launch the platform, select Analyze > Reliability and Survival > Repeated Measures Degradation. Then configure the dialog as follows:

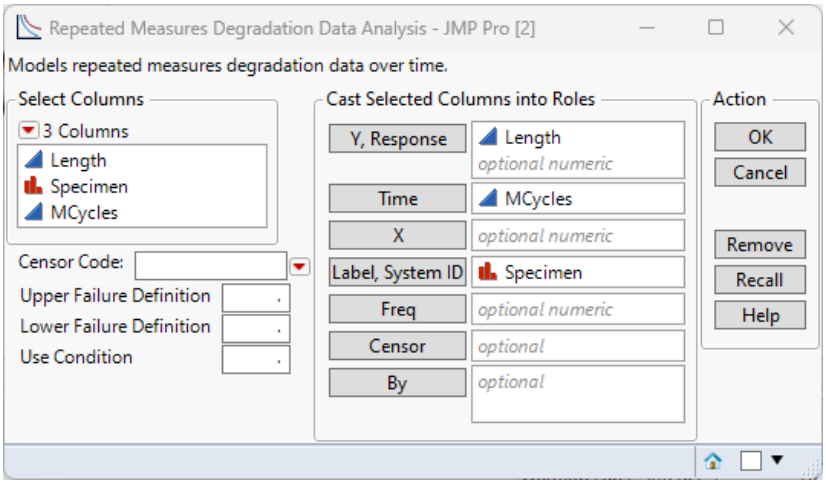


Figure 26 Repeated Measures Degradation Launch Dialog

After clicking OK, the software will produce a report as follows:

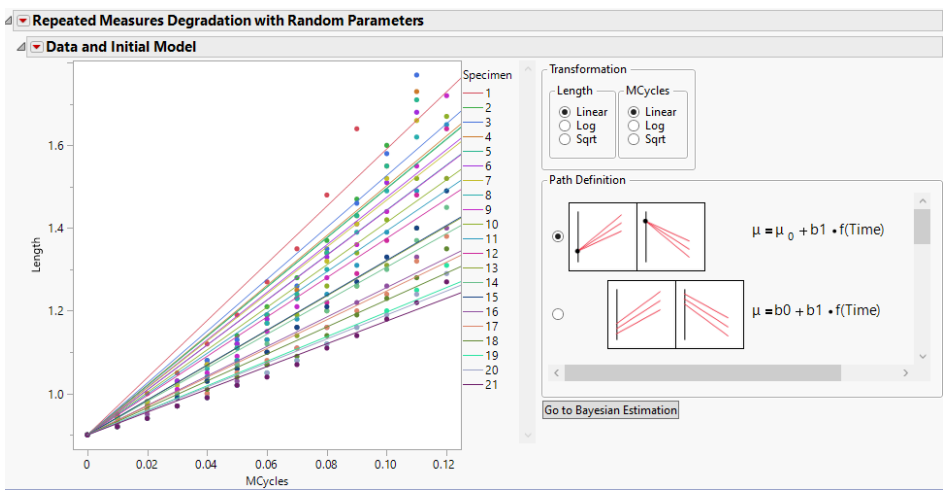


Figure 27 Repeated Measures Degradation Default Report

The default report has an overlay of data and a fitted initial model. There are a couple of panels on the top right for transforming either response or time. For our example here, that is not necessary. We don't change them. There is a "Path Definition" panel, inside of which there is a list of available models that one can choose. The first model is the simplest model, and it is the default initial model. One, however, can easily change the desired model by clicking the radio buttons in Path Definition. The Paris model is corresponding to the following radio button:

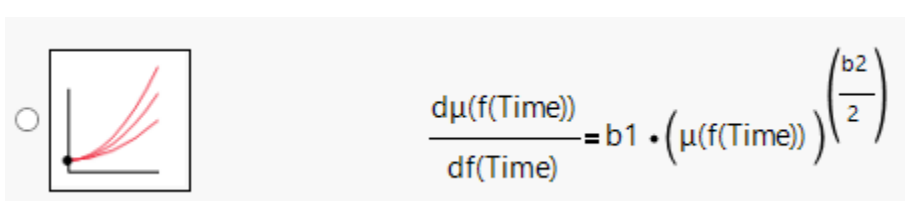


Figure 28 Paris Model Radio Button

After clicking this radio button, the plot on the left will change to what is in the following screenshot:

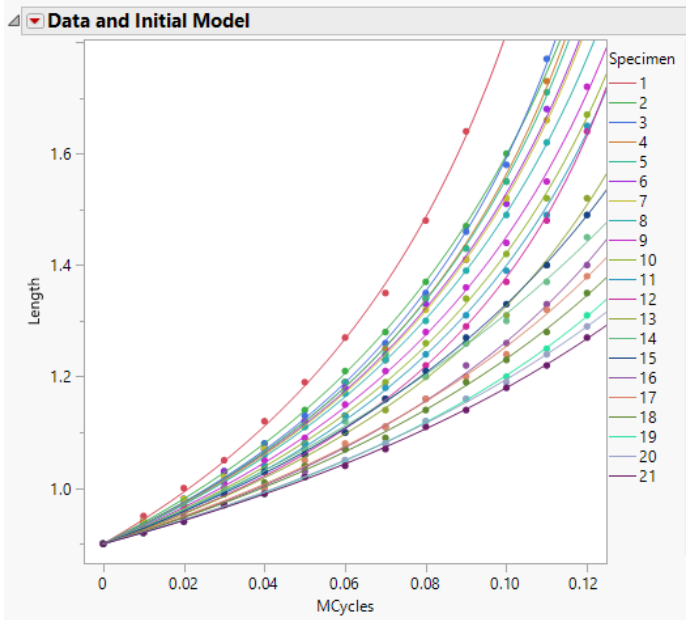


Figure 29 Initial Fit of Paris Model

The curves are the result of the initial fit of the Paris model to the data. Now click "Go to Bayesian Estimate" button below the Path Definition panel. An outline node is then appended to the report. In the outline node, there is a Formula Picture and a user interface for one to specify prior distribution and other settings. The following is the default for prior distribution specification.

Parameter	Parameter Distribution	Prior	Prior Distribution		Value		Value
b1	Normal	μ[b1]	<Normal>	Lower 99% Limit	-15.0718	Upper 99% Limit	22.50946
		σ[b1]	Half Cauchy			Scale	7.294985
b2	Normal	μ[b2]	<Normal>	Lower 99% Limit	-9.07118	Upper 99% Limit	19.47377
		σ[b2]	Half Cauchy			Scale	5.540924
Correlation		ρ[b1,b2]	Normal	Location	0	Scale	4
σ[ε]		σ[ε]	Half Cauchy			Scale	0.102426

Figure 30 Default Prior Settings for Alloy A to Fit Paris Model

For this example, with the belief that the unit parameters are bivariate normal, one only needs to click the Fit Model button not far below the prior specification. It appears that one does not even need to specify the prior distribution.

Under the hood, the software makes educated guess about the plausible range of the parameters of the prior distribution. Then expand the range to make the prior even weaker in information that it contains. Such a default prior is weakly informative and should have little influence on the posterior. And therefore, one can easily defend the choice of prior. Defending the choice of prior is probably the biggest challenge of adopting Bayesian methods in practice.

For this data and this model, with the default prior settings, it will take just a couple of minutes to finish. The software is designed to be easy to use, so it takes care of other important steps that one may not need to know. But one may need to know the effects of what are in MCMC Controls and change them when it is necessary. But let's first look at the Bayesian estimation result.

Like what we have presented, the results include a copy of prior specification and summaries of posterior samples.

Bayesian Estimates 1

Completion Time (Minute)

Priors

Parameter	Parameter Distribution	Prior	Prior Distribution		Value		Value
b1	Normal	μ[b1]	<Normal>	Lower 99% Limit	2	Upper 99% Limit	5
		σ[b1]	Half Cauchy			Scale	0.2
b2	Normal	μ[b2]	<Normal>	Lower 99% Limit	4	Upper 99% Limit	6
		σ[b2]	Half Cauchy			Scale	0.2
Correlation		ρ[b1,b2]	Normal	Location	0	Scale	4
σ[ε]		σ[ε]	Half Cauchy			Scale	3

MCMC Controls

Name	Value
Number of Monte Carlo Iterations	10000
Random Seed	32323
Warmup Laps	10
Auto Thinning	Yes
Applied Thinning	66
N Chains	1

Posterior Estimates

Population Parameters

Export Monte Carlo Samples

Export Stacked Monte Carlo Samples

Parameter	Median	95% Credible Interval Lower	95% Credible Interval Upper	Mean	Std Error
μ[b1]	3.712183	3.398352	4.021297	3.712003	0.156805
σ[b1]	0.727988	0.551954	1.016362	0.743008	0.119512
μ[b2]	5.195967	4.960578	5.422605	5.193837	0.117119
σ[b2]	0.509067	0.36237	0.743545	0.519572	0.096188
ρ[b1,b2]	-0.39774	-0.71835	0.049082	-0.38054	0.197363
σ[ε]	0.006964	0.006364	0.007659	0.006979	0.000333

Figure 31 Report of Bayesian Estimation in Repeated Measures Degradation

The posterior sampling method in this platform is Metropolis-Hastings (MH) random walk algorithm. The rejection-based sampler is not attempted because it will be guaranteed to be inefficient due to the number of parameters. Here parameters include 42 unit parameters. When one uses MH algorithm, it is important to inspect posterior samples. There are two hyperlinks by the side of the posterior summaries of population parameters. For this tutorial, we will only click the first link. And the software will generate a table with posterior samples.

The table has several embedded scripts. One should at least run the first script and produce time series plots for individual population parameters. One needs to confirm two things. First, time series should be stationary. Second, time series should have reasonably low auto correlation.

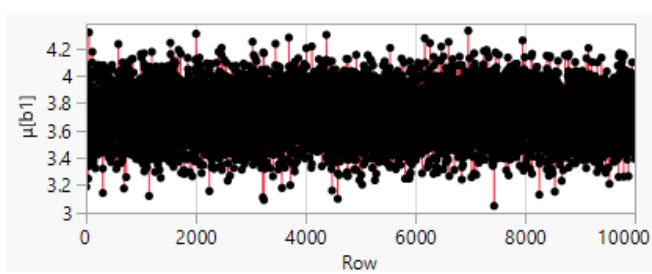
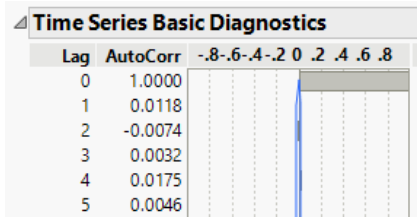


Figure 32 Stationary Time Series

By stationary, loosely speaking, one should expect time series look like a lot of random points around a horizontal line, free from any trends or patterns. Figure 32 is such an example.



Time Series Basic Diagnostics

Lag	AutoCorr
0	1.0000
1	0.0118
2	-0.0074
3	0.0032
4	0.0175
5	0.0046

Figure 33 Autocorrelation Function Report in Time Series

Meanwhile, we wish the series has as little autocorrelation as possible. Figure 33 is an example, which indicates that there is almost no autocorrelation in the series, which is the best situation we can expect.

After seeing such results, we are comfortable to proceed and make an inference about the failure distribution for this type of alloy. The definition of failure for this type of alloy is that the crack length of a specimen reaches the threshold of 1.6 inches. And the distribution of million cycles before crack reaches 1.6 inches is the failure distribution. To obtain that distribution, one should select "Show Life Distribution Profilers" from the report menu of the fitted model. And fill out the dialog accordingly. For this example, the most important item to enter is 1.6 to Upper Failure Definition. Then click OK button.

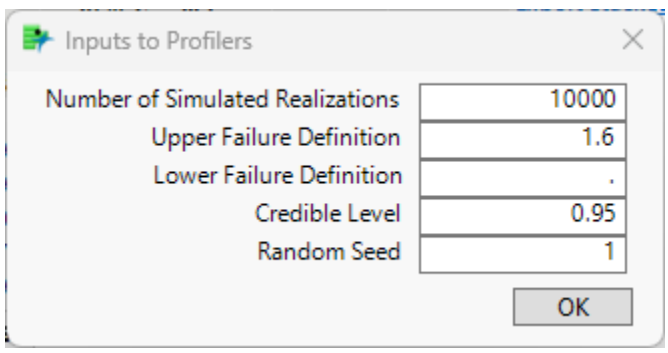


Figure 34 Alloy A Failure Definition

It will take a little time to compute, which will be based upon the posterior samples that we have obtained. In general, the time consumption is much shorter than that on drawing posterior samples. The results are two profilers. One for the failure distribution, and the other for quantile.

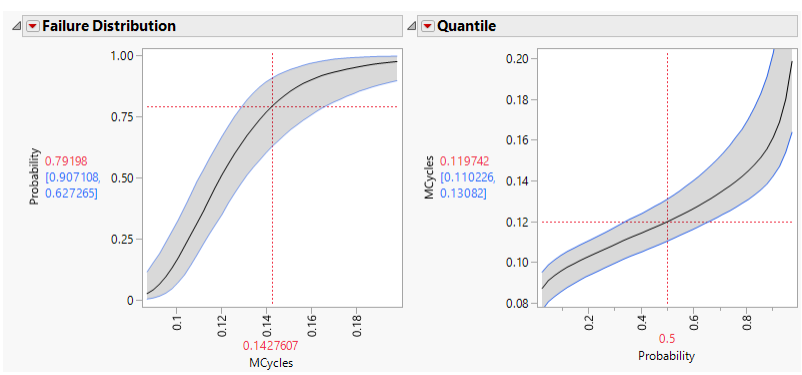


Figure 35 Failure Distribution of Alloy A

Device B

This data set is slightly more complicated than Alloy-A, because there is an acceleration factor involved, which is temperature. Figure 36 is a plot of the data.

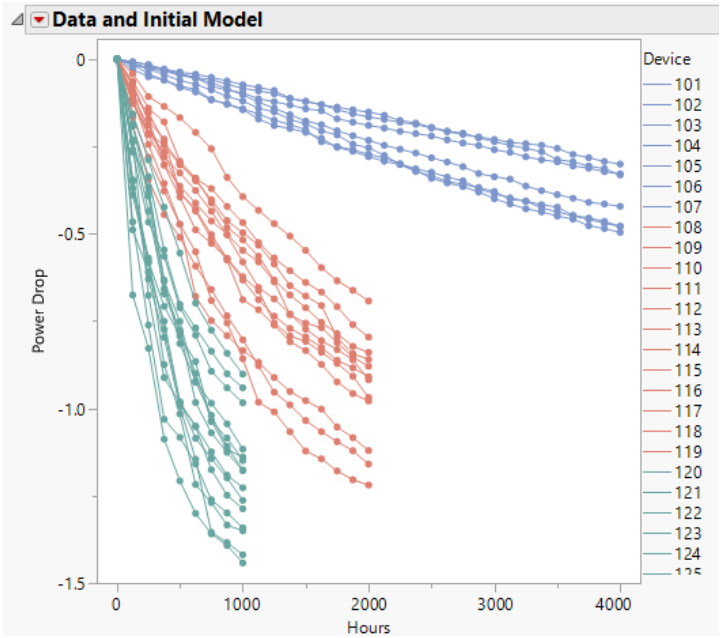


Figure 36 Device B Data

The data measures power output decrease from multiple power generators, at three temperature levels (150°C, 195°C, and 237°C). And the interest is to obtain the distribution of time in years for the power output reaches 0.5 decibels below the initial value, at the usual operating temperature at 80°C. This type of degradation data is known as the accelerated repeated measures degradation.

We already know what model to fit, but the first step is to launch the platform properly. Here is the configuration of the launch dialog for this data.

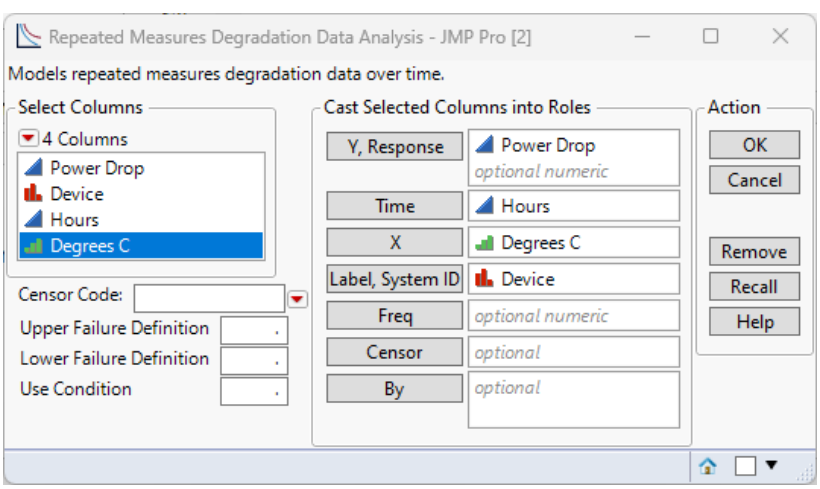
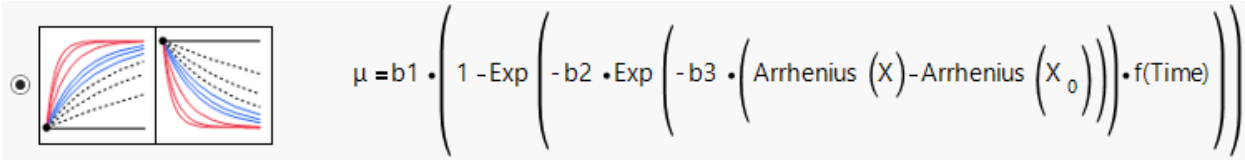


Figure 37 Repeated Measures Degradation Launch Dialog for Device B

After clicking OK, one needs to identify the desired model among the list of models in Path Definition panel. The desired model is:



By now, one should notice that the thumbnails corresponding to individual formulas in the Path Definition panel give hints whether a model is appropriate or not. Upon selecting the radio button, a dialog will appear and ask the user to determine the temperature unit. Choose Celsius for this example, then click OK. Another dialog will appear asking for a reference temperature. It suffices to accept the default for this example and click OK. After that step, click "Go to Bayesian Estimation" button to produce an outline node to fit this model.

For this example, it suffices to accept all the default settings and click "Fit Model" button to start fitting the model. It takes a little longer than what is needed for the Alloy-A data. But it is worth waiting to look at the result to learn something important. As usual, the results have a copy of the prior settings, and summaries of posterior samples. Now click the first hyperlink under the Posterior Estimates outline and generate the posterior sample table. Run the first embedded script to see time series plots of population parameters. First, one should see all series are quite stationary. Then expand "Time Series Basic Diagnostics" to see autocorrelation functions. Most series show little to no autocorrelation, except for one: b3.

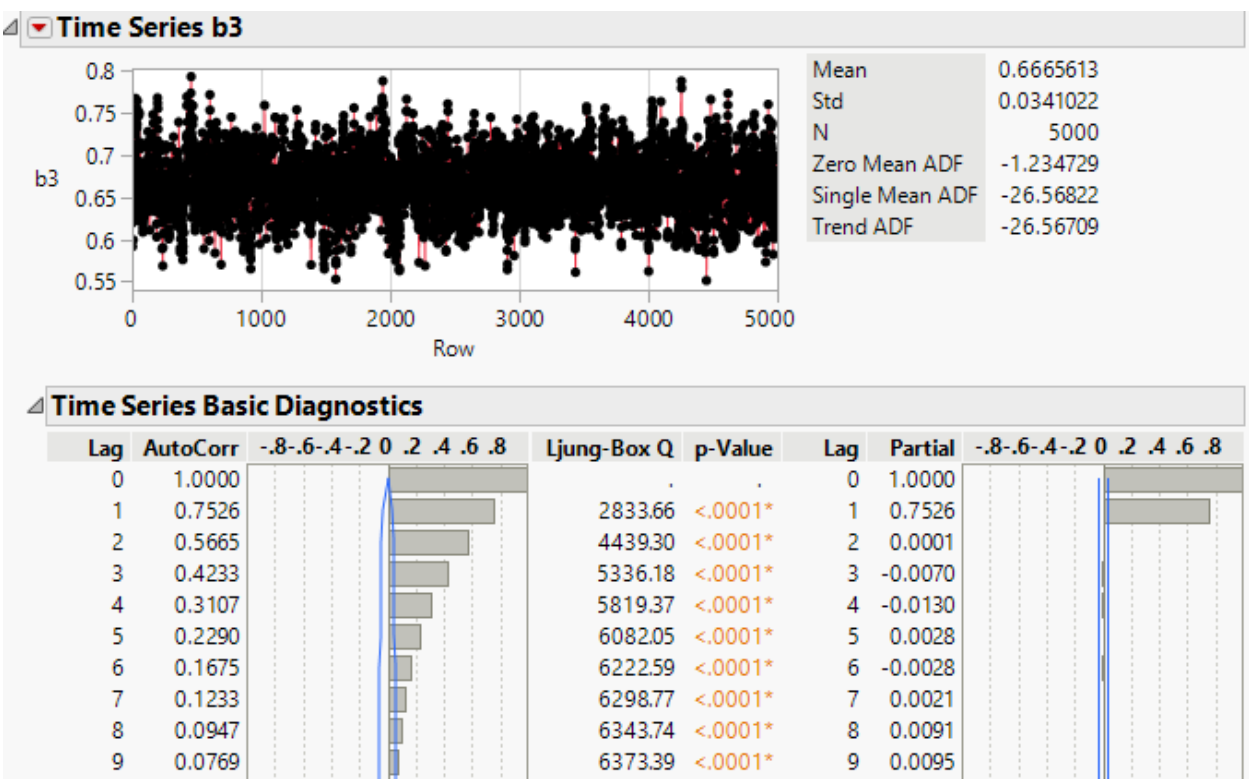


Figure 38 Autocorrelation in Posterior Samples

Here we see autocorrelation decay quickly and diminishes to zero at lag 9 or 10. This is reasonably good in the author's experience. If it decays slowly, then it is an issue. The software has already taken care of this in a clever way, but it is impossible to eliminate all of them automatically without human intervention. Now is a good time to look at what is in the MCMC Controls.

MCMC Controls

Name	Type	Value	Description
Warmup Laps	Integer	10	Set automatic tuning laps.
Auto Thinning		☑	Use suggested Thinning period.
Thinning	Integer	1	Thinning period if Auto Thinning is unchecked.
N Chains	Integer	1	Number of chains.

Figure 39 MCMC Controls for MH Algorithm in Repeated Measures Degradation

The model fitting process has two stages. The first is hyperparameter tuning, and the second is drawing posterior sample using the settings tuned in the first stage. In our implementation, the tuning process is controlled by a single number: Warmup Laps. In general, the more laps, the better the tuning result will be. But it may not be necessary to run too many warmup laps. The tuning stage will determine multiple hyperparameters that are unnecessary to expose to users, except one: Thinning. Thinning is an integer which determines how many posterior samples need to be discarded to keep autocorrelation low in posterior samples. Suppose the value of Thinning is 5. That means the algorithm will only keep one sample among every five samples. The larger the Thinning value, the more samples will be discarded, and the sampling will be slower to collect desired number of posterior samples. But on the other hand, the smaller the Thinning value, the more likely that the posterior samples will be highly autocorrelated. During tuning, the software may increase the Thinning value, but won't go beyond a thousand or so.

After explaining what Warmup Laps and Thinning affect, we can now provide suggestions on when and how to use them. One will need to change them if they see slow decay in autocorrelation, e.g. the autocorrelation remains high after lag 20 or so. The first thing one should do is to increase the Warmup Laps integer. The values 20 and 30 are reasonable values. The author has not seen the necessity to use a value larger than 50. After increasing Warmup Laps, if the autocorrelation is still high, look at "Applied Thinning" in the saved MCMC Controls. The following is from fitting Device B data. The value is 658 in this run.

MCMC Controls

Name	Value
Number of Monte Carlo Iterations	5000
Random Seed	32323
Warmup Laps	10
Auto Thinning	Yes
Applied Thinning	658
N Chains	1

Figure 40 Saved MCMC Controls

Then uncheck "Auto Thinning" and enter an integer larger than Applied Thinning in the Thinning box, which is just below the Auto Thinning check box. You may want to add a larger number aggressively but

expect to spend longer time to finish the fitting process. If it still does not appear improved, it might be a good time to contact our technical support or contact the author directly.

For this example, the autocorrelation is not an issue in the author's experience. We can proceed to make an inference about the failure time distribution estimation. Select "Show Life Distribution Profilers" from the report menu of the fitted model. And fill out the dialog accordingly. For this example, the most important items to enter include -0.5 to Lower Failure Definition, and 80 to Use Condition. Then click OK button. The generated Failure Distribution profiler gives an estimation of the failure probability at 130,000 hours (15 years) together with the credible interval; see the plot on the left in Figure 41.

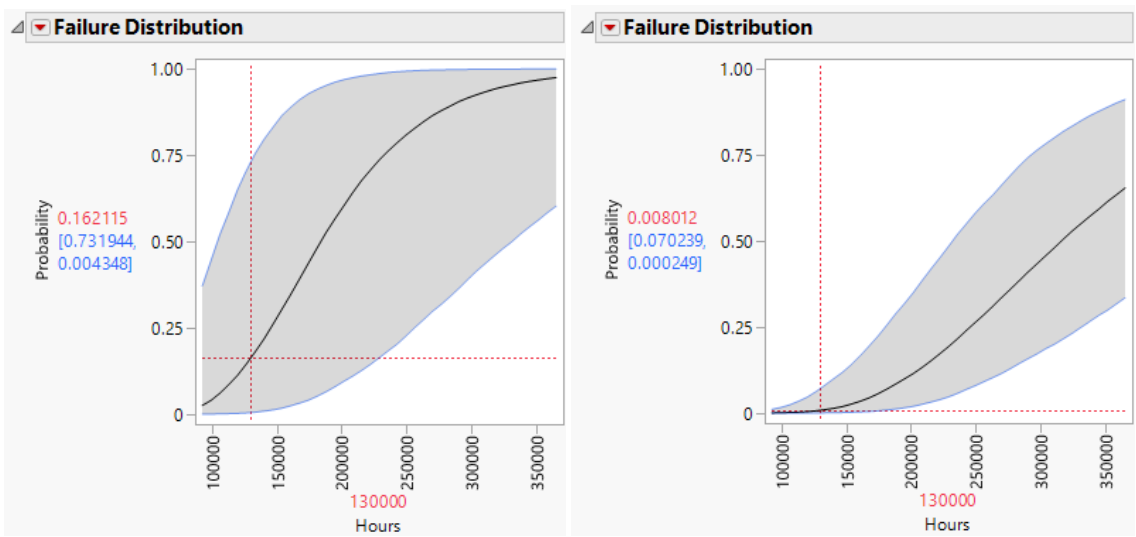


Figure 41 Failure Distribution of Device B Using Weakly Informative Prior and Informative Prior

You may notice the credible interval is wide. That is due to the amount of information that is available in the data, and the use of weakly informative prior. For this model, the parameter b_3 is known as the activation energy. In practice, engineers have prior knowledge about the quantity. In this case, engineers may apply an informative prior on the activation energy using the following setting:

Figure 42 Informative Prior on Activation Energy for Device B

Correspondingly, the Failure Distribution on the right in Figure 41 gives the estimation and credible interval. That is a substantial change. It is a trustworthy result and more useful to engineers for subsequent system reliability design work.

Conclusion

JMP reliability products are comprehensive. What we have explored and demonstrated here is the tip of the iceberg. We hope that this tutorial leaves you an impression about the wide range of capabilities of JMP reliability products.

The Bayesian methods that we have implemented are at the frontier of reliability data analysis research. There is still much to do in the future. We hope what we have put in JMP provide the engineers and scientists an easy access to solve some common problems that need Bayesian methods.

Acknowledge

The achievements of JMP reliability platforms rely on the dedication from multiple people besides the author. Here is a list of them who have contributed directly and served for the longest time. William Meeker for the guidance in the past fifteen years on the development of JMP reliability platforms. John Sall for initiating the development many years ago and continuingly overseeing the development. Bradley Jones for hiring the author and initiating the expansion of modern development. Christopher Goltwalt for the initial numerical algorithm development for the expansion and supervision of the expansion for the last fifteen years. Leo Wright for his product management role in the last fifteen years on working with industry thought leaders, prioritizing the development and overseeing the marketing of the reliability products. Rajneesh Rajneesh for testing all my development work. Laura Archer for providing technical support. Tom Donnelly for promoting reliability products in all sales events and among important communities.

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