On the Validation of Statistical Software

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Outline

- What is Software Testing?
- The Testing Challenge
- Test Selection as a Designed Experiment
- Covering Arrays

What is software testing?

- To show the software works as intended.
- To ensure there are no bugs in the software.
- Assures us the software does what it is supposed to do.

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- ~~Assures us the software does what it is supposed to do.~~

These approach the problem as showing the software works, leading us down this path.

Consider

- “Can you check that this works?”

Vs.

- “Try and break it.”

What is software testing?

"Testing is the process of executing a program with the intent of finding errors."

G. Myers, The Art of Software Testing, Wiley, 1979

Where are the bugs?

“Bugs lurk in corners and congregate at boundaries.”

B. Beizer, Software Testing Techniques, Van Nostrand Reinhold, 1983

The testing challenge

1. Selection problem: How do you select test cases from the input space of the system so that the chance of finding faults, while staying within budget, is maximized?
2. Quality problem: Can you make informed assertions about “fitness for use” as testing unfolds?
3. Oracle problem: How do you determine expected behavior for each test case?

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The oracle problem

How to determine if a test outcome is what was expected

- Discussion of statistical software usually focused on numerical accuracy
- We often want to go further than this
- How to determine the expected outcome?
- What does it mean if the results change?

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The oracle problem

What is the expected result?

- Software crash
- Timing
- Visualization
- Set of statistics
- Numerics (how close is good enough?)
- And so on…

- May not be existing packages that the test engineer can use

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Common techniques

Repeated running of unit tests as a regression test suite.

Unit Testing

Develop test cases for validating the smallest testable component (i.e., a unit) of a software package before focusing on the overall integrated system.

Regression Testing

Ensure the software still works as intended after a change.

Test selection via Design of Experiments (DOE)

- Given a set of inputs, how to test these effectively and efficiently
- Deterministic*
- Pass/Fail - based on oracle
- Fault detection, rather than model fitting
  - Looking for failure-inducing combinations
- What is a good design (test suite)?
  - “Bugs lurk in corners and congregate at boundaries.”

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Fundamental principles of factorial effects

Wu & Hamada (2011)

Effect hierarchy - *i)* Lower order effects are more likely to be important than higher order effects. *ii)* Effects of the same order are equally likely to be important.

Effect sparsity - The number of relatively important effects will be small.

Effect heredity - An interaction is significant only if at least one of the parent factors involving the interaction is significant.

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- What if we think of failure-inducing combinations as important effects?

Fundamental principles of input combinations

Lekivetz & Morgan (2020)

Combination hierarchy: *i)* Combinations involving fewer inputs are more likely to be failure-inducing than those involving more inputs.

ii) Combinations of the same order are equally likely to be important.

Combination sparsity: The number of failure-inducing combinations will be small.

Combination heredity: A combination is *more likely* failure-inducing if at least one of the parent factors involving the interaction is known to be more likely involved in inducing failures.

Empirical Evidence

#factors involved in failure	Medical devices	Browser	Server	NASA GSFC
1	66	29	42	68
2	97	76	70	93
3	99	95	89	98
4	100	97	96	100
5		99	96	
6		100	100	

Cumulative percentage of faults in software systems triggered by interactions involving number of factors indicated in leftmost column (Kuhn et al., 2004).

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Combinatorial testing

"Bugs lurk in corners and congregate at boundaries."

A family of test case selection strategies used to test complex engineered systems. For a complex engineered system with m inputs, such as a software system, a strength t covering array will ensure that all possible combinations of the values for any set of $t \leq m$ inputs will appear in the derived test suite at least once.

- Solves the selection problem and the quality problem
  - A way to select cases and assert what has been tested (all combinations involving up to t inputs) – "pseudo-exhaustive"
  - Moves beyond one-factor-at-a-time (OFAT) - unit tests as the simplest units

Covering Arrays

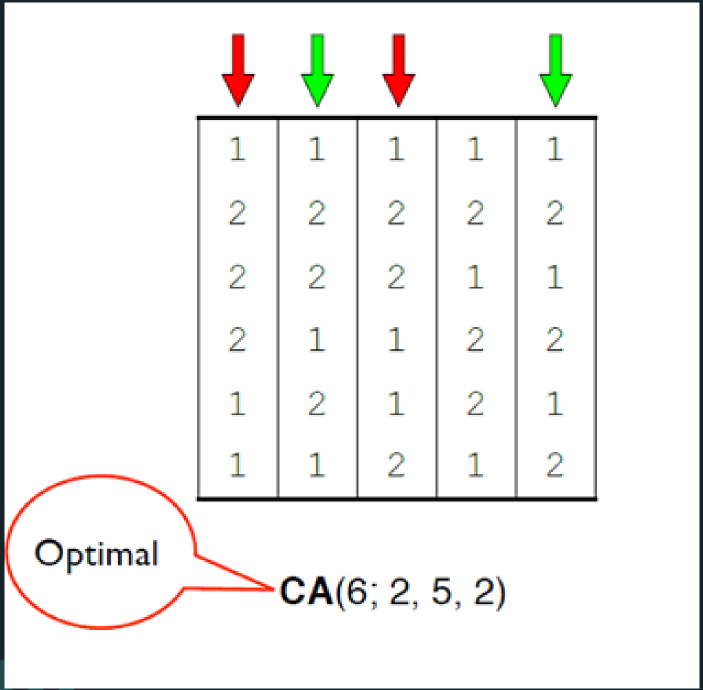
***Covering Arrays*:** For a set of factors, a t-covering array (or strength t) has the property that for any subset of t factors, every possible combination of levels occurs *at least once*.

Orthogonal Arrays: Every possible combination occurs the same number of times.

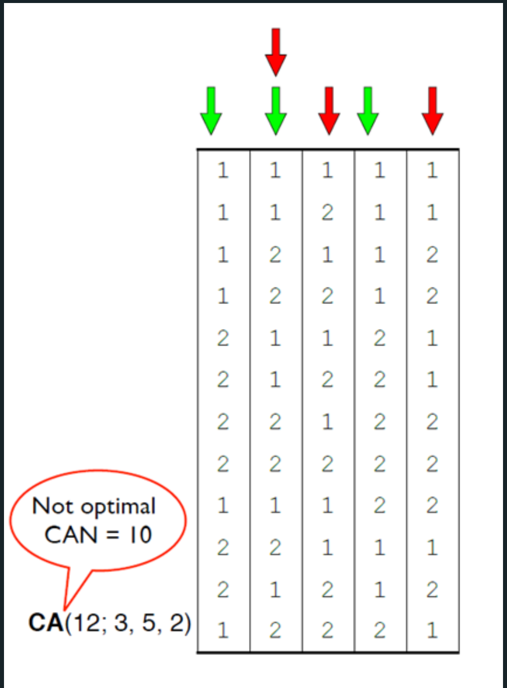
N	4	5	6	7	8	9	10	11	12	13	14
m	3	4	10	15	35	56	126	210	462	792	1716

CA(N; 2, 2^m), m ≤ 1716

Examples



1	1	1	1	1
2	2	2	2	2
2	2	2	1	1
2	1	1	2	2
1	2	1	2	1
1	1	2	1	2



1	1	1	1	1
1	1	2	1	1
1	2	1	1	2
1	2	2	1	2
2	1	1	2	1
2	1	2	2	1
2	2	1	2	2
2	2	2	2	2
1	1	1	2	2
2	2	1	1	1
2	1	2	1	2
1	2	2	2	1

Why Covering Arrays?

- Cost-efficient
- Selection problem – what to test (DOE)
- Quality problem - if all tests pass a strength *t* covering array, can ascertain there are no faults due to *t**-factor combinations *(t*<=t)*
- Disciplined approach to testing
- Another tool in the tool chest
- When do combination sparsity and hierarchy hold?

System under test (SUT)

i.e. the software

Covering Array	SUT
Columns	Inputs
Levels per column	Equivalence classes* per input
Strength (i.e. *t*)	Testing budget
Rows	Test cases

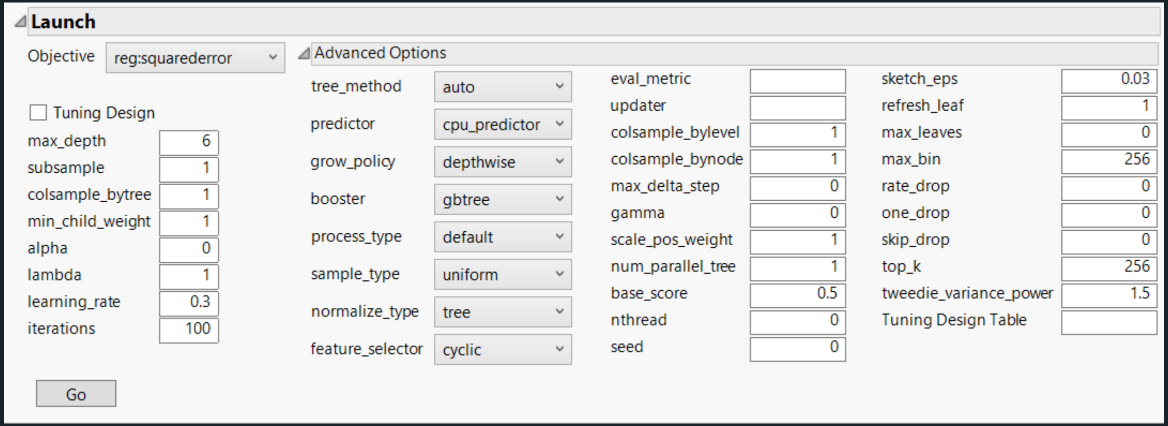
Other Considerations

- Fault localization – find the underlying cause when you have a failure.
  - Not as easy as it seems
- Constraints (disallowed combinations/forbidden edges)
- Locating / Detecting Arrays
- Random seeds (make sure to keep them)
- Random inputs/equivalence partitioning/boundary-value analysis
- Data set generation*
- Software (ACTS, JMP Pro, Hexawise, Testcover.com, etc…)

XGBoost

What is XGBoost?

“An optimized distributed gradient boosting library designed to be highly efficient, flexible and portable.”



T. Chen and C. Guestrin. 2016. XGBoost: A Scalable Tree Boosting System.

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XGBoost Selected Inputs

	Hyperparameter	Type
1	Max_depth	*continuous*
2	Subsample	*continuous*
3	Colsample_bytree	*continuous*
4	Min_child_weight	*continuous*
5	Alpha	*continuous*
6	Lambda	*continuous*
7	Learning_rate	*continuous*
8	Iterations	*continuous*
9	Tree_method	*categorical (6)*
10	Predictor	*categorical (2)*
11	Grow_policy	*categorical (2)*
12	Booster	*categorical (3)*
13	Process_type	*categorical (2)*
14	Sample_type	*categorical (2)*
15	Feature_selector	*categorical (4)*
16	Colsample_bylevel	*continuous*
17	Colsample_bynode	*continuous*
18	Max_delta_step	*continuous*

	Hyperparameter	Type
19	Gamma	*continuous*
20	Scale_pos_weight	*continuous*
21	Num_parallel_tree	*continuous*
22	Base_score	*continuous*
23	Nthread	*continuous*
24	Seed	*continuous*
25	Sketch_eps	*continuous*
26	Refresh_leaf	*continuous*
27	Max_leaves	*continuous*
28	Max_bin	*continuous*
29	Rate_drop	*continuous*
30	One_drop	*continuous*
31	Skip_drop	*continuous*
32	Top_k	*continuous*
33	Tweedie_variance_power	*continuous*
34	Normalize_type	*categorical (2)*

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XGBoost Selected Inputs

	Input	# of levels
1	Max_depth	3
2	Subsample	3
3	Colsample_bytree	3
4	Min_child_weight	3
5	Alpha	3
6	Lambda	3
7	Learning_rate	3
8	Iterations	3
9	Tree_method	6
10	Predictor	2
11	Grow_policy	2
12	Booster	3
13	Process_type	2
14	Sample_type	2
15	Feature_selector	4
16	Colsample_bylevel	3
17	Colsample_bynode	3
18	Max_delta_step	3

	Input	# of levels
19	Gamma	3
20	Scale_pos_weight	3
21	Num_parallel_tree	3
22	Base_score	3
23	Nthread	3
24	Seed	3
25	Sketch_eps	3
26	Refresh_leaf	2
27	Max_leaves	3
28	Max_bin	3
29	Rate_drop	3
30	One_drop	2
31	Skip_drop	3
32	Top_k	3
33	Tweedie_variance_power	3
34	Normalize_type	2

Results

- Input space: $6*4*3^{25}*2^{7}$ = 2,602,870,608,208,896 points
- Strength 2 CA can be constructed in 25 runs
  - 72% 3-coverage
  - 35% 4-coverage
- Strength 3 CA can be constructed in 150 runs
  - 90% 4-coverage

Thanks!

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