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Regression and Time Series Mixture Approaches to Predict Resilience

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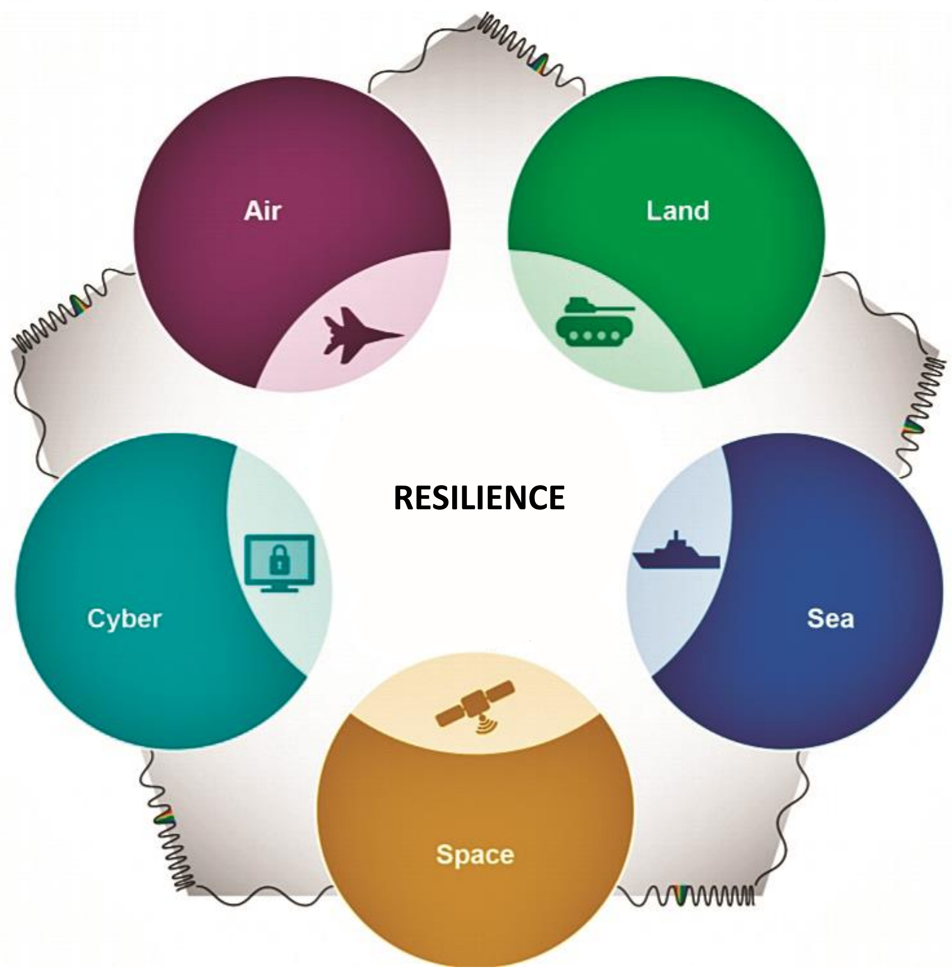
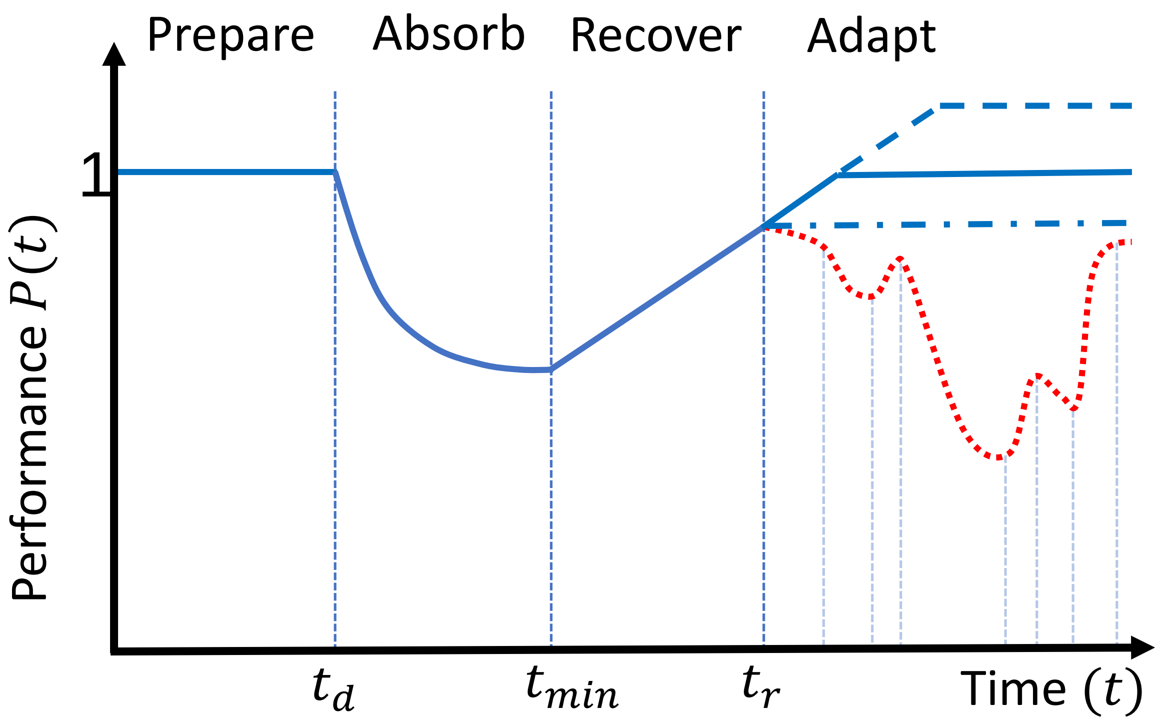
Introduction

- **System Resilience**
  - Ability to recover from failures
- **Lack of resilience in critical technologies is dangerous**
  - Disrupt military logistics
  - Affect DoD's ability to maintain operations
  - Lead to schedule and budget overruns
  - Endanger military and civilian lives
- **Common Practices**
  - Quantitative resilience metrics
- **Contributions**
  - Mathematical models to track and predict change in performance



System Resilience



Real world disaster management does not look like a textbook curve

- Performance (P) is
  - Domain dependent
  - The level of goal achievement of a system or task

$$P(i) = P(i-1) + \Delta P(i)$$

where

$P(i)$ = performance in present time interval i
$P(i-1)$ = performance in previous time interval $(i-1)$
$\Delta P(i)$ = change in performance (assumed I.I.D.)

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Resilience Modeling Approaches

- Modeling change in performance (ΔP)

- **Regression Models**
  - Multiple linear regression (MLR), (MLRI), and (PR)

$$\Delta\hat{P}(i)_{MLR} = \beta_0 + \sum_{j=1}^{m} \beta_j X_j\,(i)$$

m = number of covariates
β_0 = baseline change in performance
β_j = coefficients of disruptions and activities
X_j = covariates driving degradation/recovery
ℓ = number of lags

- **Time Series Models**
  - Multivariate Vector Auto-Regressive (MVAR), and (MVARMA)

$$\Delta\hat{P}(i)_{MVAR} = \beta_0 + \sum_{k=1}^{\ell} \beta_k P(i-k) + \sum_{j=1}^{m} \sum_{k=1}^{\ell} \beta_{j(\ell+k)} X_j\,(i-k)$$

Mixture Resilience Models

- Modeling change in performance (ΔP)
  - Multiple linear regression and Multivariate Vector Auto-Regressive (MLR-MVAR)

$$\Delta\hat{P}(i)_{MLR-MVAR} = \beta_0 + \sum_{j=1}^{m} \beta_j X_j(i) + \sum_{k=1}^{\ell} \phi_k P(i-k) + \sum_{j=1}^{m} \sum_{k=1}^{\ell} \phi_{j(\ell+k)} X_j(i-k)$$

  - MLR-MVARMA
  - MLRI-MVAR
  - MLRI-MVARMA
  - PR-MVAR
  - PR-MVARMA

Steps to Apply Resilience Models

1) Identify disruptive and restorative activities X_j

2) Collect data

3) Estimate model parameters β_0 and β_j

- Maximum Likelihood Estimation (MLE)

4) Validate model with statistical measures

- RMSE – Root Mean Squares Error
- PMSE – Predictive Mean Squares Error
- r_{adj}^2 – Adjusted coefficient of determination
- Confidence Intervals

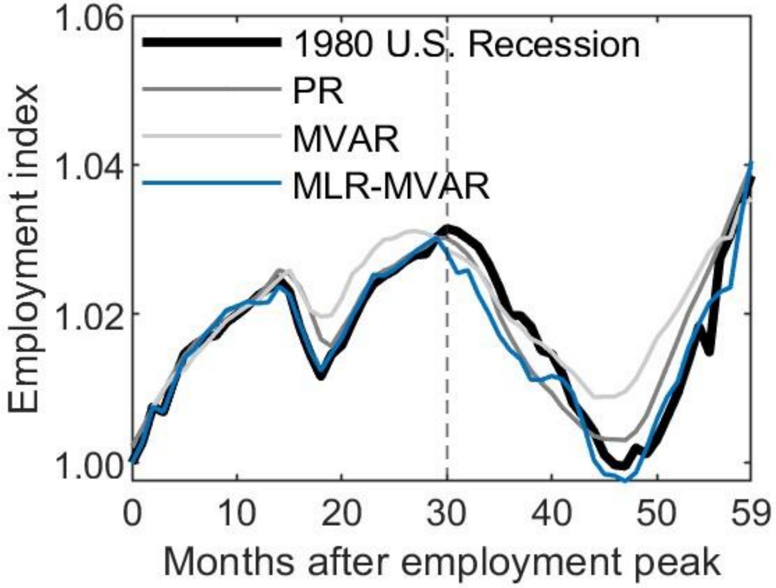
Illustration: 1980 US Recession – Energy Crisis

- Performance $P(i)$ is the number of employments in the US
  - Engineering data was not available
- Covariates in bold are selected by forward and backward stepwise procedures

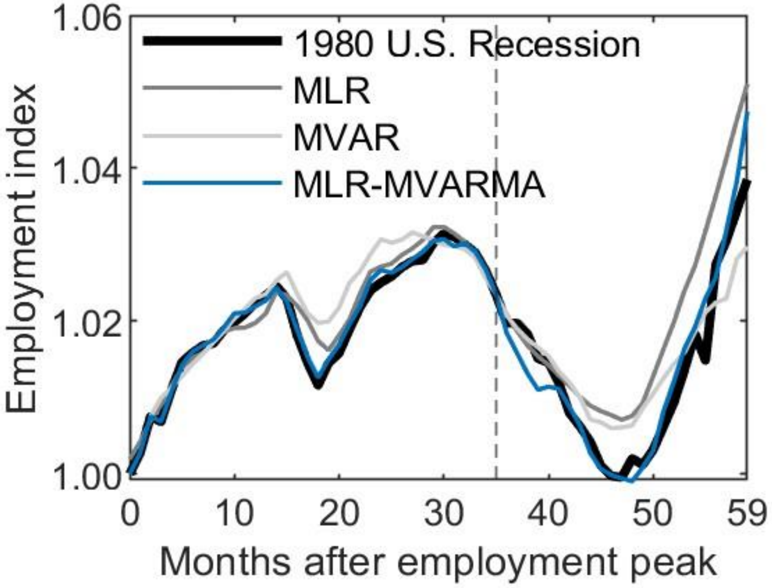
Covariates	Description	Covariates	Description
X1	Treasury Yield Curve	X5	Personal Consumption Expenditures
X2	Industrial Production	X6	S&P 500 Index Stock Market
X3	Federal Funds Rate	X7	Consumer Price Index
X4	Mortgage Rate	X8	Crude Oil Prices

Illustration: Computation Steps

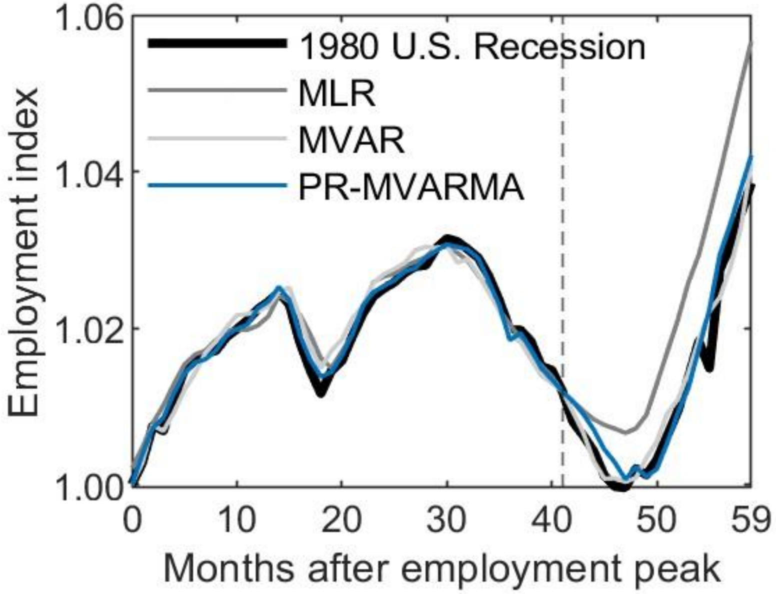
- 11 Models were tested: 3 regressions, 2 time series, 6 mixture models
  - Different combinations of covariates and lags for each approach
- 4 data subsets were used for model fitting: 50%, 60%, 70%, 80%
- Model fits and goodness-of-fit were computed
- The best model fit from each approach for each subset considered is plotted against each other for analysis
- The best model fit overall is picked for further analysis



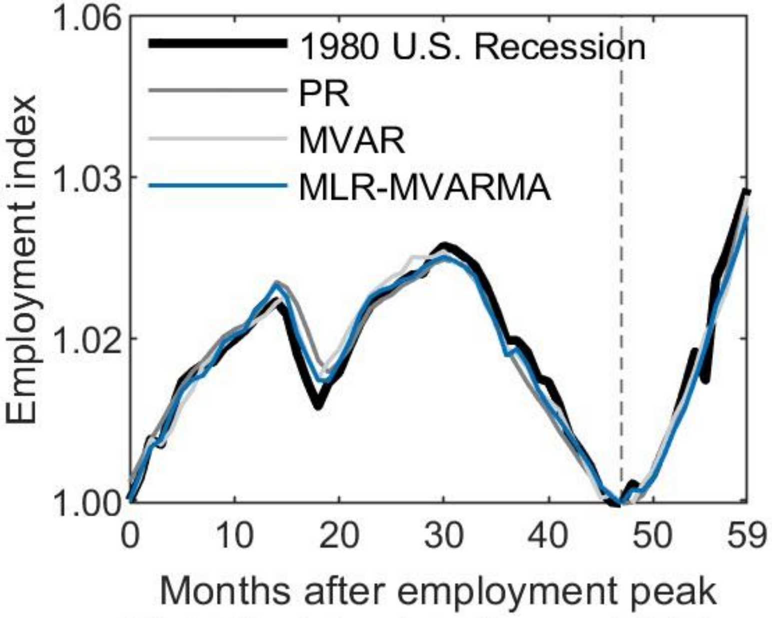
(a) 50% of the data for model fitting



(b) 60% of the data for model fitting

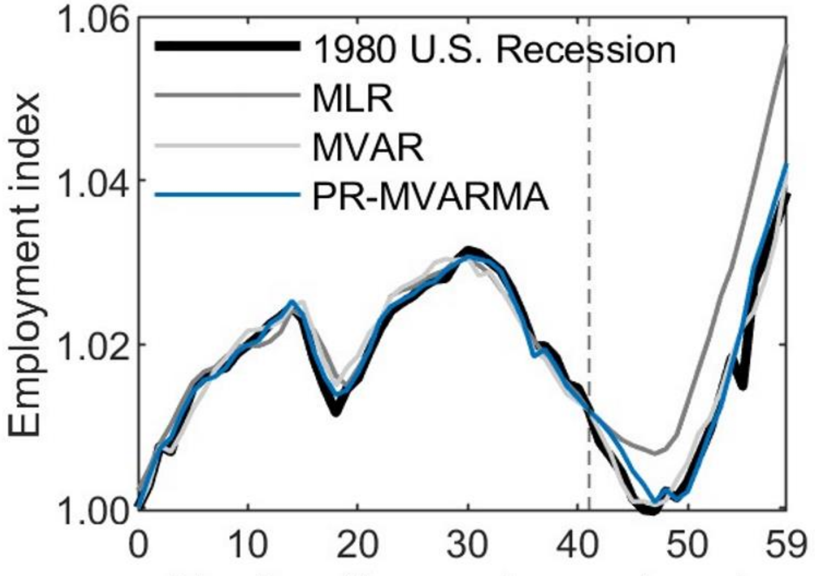


(c) 70% of the data for model fitting



(d) 80% of the data for model fitting

Illustration: Validation



(c) 70% of the data for model fitting

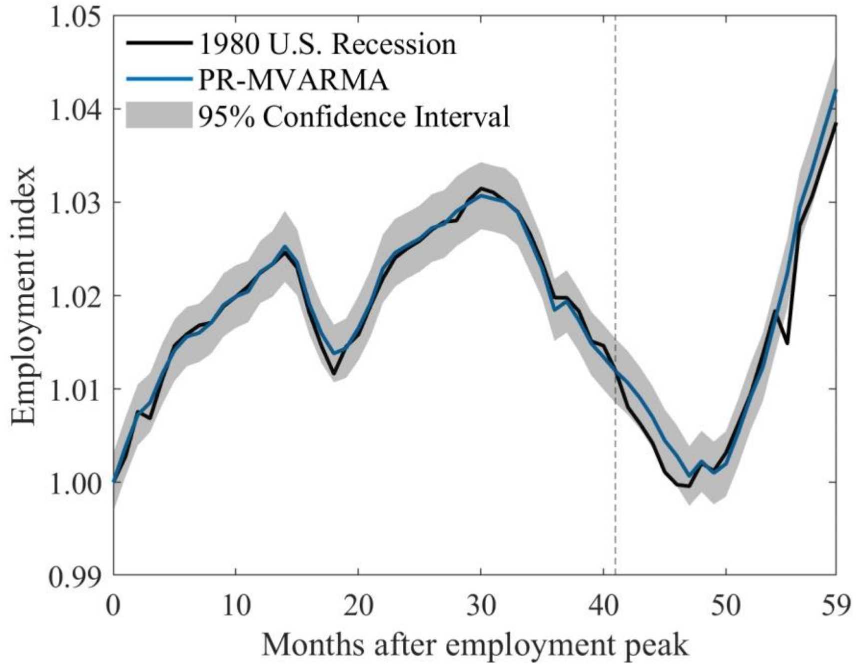
Goodness-of-fit measures

Model	Covariates	Lags	Param.	RMSE	PRMSE	r^2_{adj}
MLR	X8, X3	0	3	0.00524	0.00855	0.93122
MVAR	X8, X3, X5	4	17	0.00235	0.00240	0.92277
PR-MVARMA	X8, X3, X5, X2, X6, X4	1	21	**0.00192**	**0.00231**	**0.98628**

Mixture models predict system performance more accurately

Illustration: Best Model Overall

- Polynomial Regression and Multivariate Vector Auto-Regressive Moving Average (PR-MVARMA)



Conclusion

- **This talk presented**
  - Mathematical modeling approaches to track and predict system resilience
    - Informing systematic quantitative tests and evaluation
- **Results suggest that**
  - Regression and time series models fit well resilience curves
  - Mixture models characterize better small perturbations
    - Improving tracking and prediction abilities
    - Demonstrating superior performance in long-term predictions
- **Future research**
  - Optimal allocation of activities to achieve performance threshold on timeline
  - Development of a predictive system resilience tool

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